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# Strategic Electoral Rule Choice Under Uncertainty

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## **Abstract**

We study electoral rule choice in a multi-party model with office-motivated parties and electoral outcome uncertainty. We show that when all dominant parties (parties with positive probability of winning the elections) have sufficiently good chances of winning, they agree to change the PR-rule with a more majoritarian one in order to increase their chances of forming a single-party government. We identify the exact degree of disproportionality of the new rule and we prove that it is increasing in the expected vote share of the smaller parties (parties with zero probability of winning). The necessary and sufficient conditions for such collusion in favor of a majoritarian rule are: a) the high rents from a single-party government, b) sufficient uncertainty over the electoral outcome. Our theoretical predictions regarding the degree of the disproportionality of the electoral rule are supported by empirical evidence.

*Keywords:* electoral rules; disproportionality; seat premium; single-party government; uncertainty; strategic coordination

*JEL classification:* D02, D72, H10

# 1 Introduction

Electoral rules, beyond their effects on various aspects of social welfare (Hummel 2011; Persson, Roland and Tabellini 2007; Iversen and Soskice 2006), also affect the allocation of office rents among competing parties. This is so, because the distribution of parliamentary seats among parties is a significant determinant of relative political power (Blais 1991). As Duverger’s (1954) law postulates, a *proportional* rule that favors multi-partism will most likely result in a multi-party coalition government being formed. Therefore, the spoils of the office have to be shared among many parties. In contrast, under a more *majoritarian* rule (e.g., first-past-the-post) the winning party can more easily form a single-party government, thus being able to enjoy the spoils of office alone and advance its policy agenda without having to compromise (Tsebelis 1999).

Since parties’ payoffs depend on the applied electoral rule, it follows that political parties have incentives to *strategically* choose an electoral rule which best serves their interests. That is, going in the opposite direction, it is reasonable to expect that electoral rules themselves are also the result of strategic decisions made by parties. For instance, Colomer (2005) argues that existing political parties tend to choose electoral systems that allow them to “consolidate and reinforce the current party-system instead of changing it dramatically” and concludes that political systems that are dominated by few parties tend to establish majoritarian electoral rules.

Boix (1999) makes a similar point, suggesting that the existing variation in electoral rules across many parliamentary democracies is due to the strategic decisions that ruling parties make in order to maximize their representation in the legislature. Boix (1999) claims that if the electoral competition is less uncertain and the existing electoral rule serves the current ruling parties then, the status quo is maintained as parties have limited incentives to modify the electoral system. However, if the degree of uncertainty increases the ruling parties will consider changing the current electoral rule depending on two conditions: Firstly, the strength of the other parties, and secondly, the coordinating capabilities of the dominant parties.

In the same vein, Benoit (2004) studies the choice of electoral rules by rational, seat-maximizing parties and codifies the patterns of electoral rule change in various party-systems. According to this framework, electoral rule change occurs endogenously when two conditions are met. First, it must be that *all parties* favoring the reform should be expected to score seat gains under the new rule. Second, those parties should be able to muster enough votes in the current parliament in order to enact this change.

But, of course, it need not be that all parties share the same preferences over electoral rules. For

example small parties might prefer more proportional electoral rules to less permissive ones, since their seat share under the latter is smaller than their vote share and the probability that they participate in a government is very small since such systems regularly result in *single-party* governments.<sup>1</sup> But what about larger (or else dominant) parties? Can it be that all of them prefer a more *disproportional* (less permissive) rule, or if one dominant party prefers a more disproportional rule (possibly because it is the most likely winner of forthcoming elections) then the other dominant parties prefer a more proportional one? That is, what are the incentives which can trigger this type of strategic coordination among dominant parties in adopting a less permissive electoral rule and how are those incentives aligned in the presence of electoral uncertainty in order to enable collusion?

The answer to these questions can help us to better understand both the rise in number of major electoral reforms observed in the last two decades and also the increasingly less permissive (more majoritarian) nature of many of those reforms (Table 1). Until recently, the number of major electoral reforms in advanced western democracies was particularly low and confined to some outlier countries (e.g., Greece). For instance, Katz (2005) documents only fourteen incidents of major electoral reforms from the end of World War II until the early 1990s. Moreover, the majority of electoral reforms that took place since the beginning of the twentieth century were permissive in nature (Colomer 2004). Yet, after the early 1990s the tide seems to have turned decisively, especially in those newly established democracies in east Europe (Table 1). Not only is the occurrence of majoritarian reforms more frequent than it used to be in the past, but at the same time majoritarian reforms appear to be the new norm.<sup>2</sup> Given that those type of reforms used to be rare in the past, what has changed during the last couple of decades and the frequency of majoritarian reforms has increased over time? Furthermore, since in many of those instances (e.g., Italy in 1999 and 2014; Taiwan in 2005; Lithuania in 2000; Poland in 1993; Romania in 2008 and 2012; Greece in 2004) the majoritarian reforms have been sponsored by both dominant parties, what drives this type of strategic coordination between dominant parties in choosing more majoritarian electoral rules?

[Insert Table 1 about here]

In order to address these new empirical regularities, we formalize the idea that it is parties that choose the electoral rule. Building on existing literature of endogenous electoral rule choice, we consider a party-system with two *dominant* parties (parties which have positive probability of winning the elections) where

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<sup>1</sup>One exception can be the case of nationalist parties (e.g., Sinn Fein or the Scottish nationalists) which have territorially concentrated electoral support and hence, they might also prefer the simple plurality rule.

<sup>2</sup>Riera (2013) also records 128 cases of major and minor electoral reforms from 1945 to 2010. More than a third of them (46) were unambiguously majoritarian in nature. Moreover, two-thirds of those majoritarian reforms took place after 1990 in those newly established democracies.

each party cares for its parliamentary seat share and for office rents and where there is *uncertainty* over the electoral outcome.<sup>3</sup> In such a framework, we model electoral rule choice as a strategic decision made by the parties through a parliamentary voting procedure and we study whether incentives of dominant parties (as far as electoral reforms are concerned) can be aligned, thus enabling their coordination in altering the rules of the game. Assuming that the current rule is a proportional one,<sup>4</sup> we show that dominant parties will agree in adopting a majoritarian rule if each of the two dominant parties has sufficiently good chances of being the plurality winner of the next elections. We moreover characterize the exact degree of disproportionality of this majoritarian rule: it is such that the plurality winner can always form a single-party government. That is, the two dominant parties introduce a majoritarian rule which eliminates the role that smaller parties would have otherwise played in a coalition government. What is perhaps more important is that the degree to which the two dominant parties distort the proportional rule is increasing in the expected cumulative vote share of smaller parties, or else, in the effective number of parties. The implications of our model find strong support in our empirical and case-studies analyses.

We identify three main forces which are behind these results. First, parties' strict preference for a single-party government implies that an additional parliamentary seat increases their utility via two channels. On the one hand, it can increase their bargaining power within a coalition, the number of ministerial portfolios they hold and their control over the legislative agenda. On the other hand, it brings them closer to the parliamentary majority threshold which they need to surpass in order to form a single-party government and enjoy the perks of holding office exclusively. That is, their desire for more seats serves a greater end: it increases their chances of forming a single-party government. Hence, the marginal utility of an extra seat is smaller when a party already commands the absolute majority in the parliament, as the second channel disappears.

The second element is the introduction of electoral uncertainty. Uncertainty plays a key role in our model by enabling collusion among dominant parties. In particular, when elections are *competitive*, dominant parties might have a common interest to coordinate and adopt a less permissive (more majoritarian) electoral rule in order to consolidate the status quo. The intuition is simple: without competitive elections, the sure loser will never agree to a more majoritarian electoral rule, since its chances of forming a

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<sup>3</sup>As in Andrews and Jackman (2005), the term *electoral uncertainty* implies that elections are competitive and contested by both dominant parties.

<sup>4</sup>Transitions from a majoritarian rule to a more proportional do not usually require any strategic coordination. Consider the following case: under a majoritarian rule (such as FPTP) it is very likely that a single party commands the majority in the parliament. Hence, if this party is expected to be the winner of the election the majoritarian rule is maintained (or even reinforced). On the other hand, if the party is expected to lose and has the necessary votes to enact a reform a more permissive electoral rule will be introduced. In both cases, electoral rule reform is the outcome of a single-agent payoff maximization programme without involving any strategic interaction (see Appendix B and also Ergun 2010).

single-party government are zero. It is their strict preference for *single-party governments* in conjunction with *electoral uncertainty* that triggers collusion, as we show.<sup>5</sup> The latter, unlike Boix (1999) who claims that electoral uncertainty prevents coordination, is found to be a *sine qua non* condition that aligns the incentives of the two dominant parties, therefore enabling strategic coordination. Without formally modelling electoral uncertainty it would have been impossible to explain the recent wave of transitions from a more to a less permissive rule as the sure losers would always have incentives to block such reforms.

But, in addition to facilitating the strategic coordination among dominant parties, the introduction of electoral uncertainty serves a more important purpose. It allows our model to address the substantive historical implications of electoral reforms and speak to recent patterns and episodes of electoral rule change. For it is the introduction of electoral uncertainty that allows us to focus on those relatively more rare, but strategically non-trivial and thus more interesting, electoral reforms such as the recent majoritarian transitions in east Europe and elsewhere. This is of particular interest, since most of the recent majoritarian transitions took place in new democracies where admittedly electoral uncertainty is expected to be higher as the political environment is still volatile. Moreover, unlike many mature democracies where major electoral reforms are less frequent (Katz 2005), experimentation over the choice of electoral rules is more likely to occur in those countries that have experienced a recent democratic transition and need to design those institutions anew. Hence, our choice to explicitly introduce electoral uncertainty not only makes our model relevant for the study of the most recent (and more challenging) episodes of electoral rule reform but, at the same time, it also highlights the mechanism (strategic coordination) which was behind many of those majoritarian transitions.

The final element is to consider a continuum of electoral rules that capture all possible degrees of disproportionality (from pure PR to FPTP with a unique nation-wide district).<sup>6</sup> To this aim we introduce in the theoretical modeling the *seat premium* system. This rule allocates a fraction of the seats according to PR while, the rest are given to the *simple plurality* winner as a premium.<sup>7</sup> Hence, by varying the amount of the premium we can simulate electoral rules with different degrees of disproportionality. This

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<sup>5</sup>Here, we need to clarify that in our set-up, parties in favor of electoral rule reform, need only have *expected* utility gains and not necessarily realized *ex post* seat gains (as in Benoit 2004). This is so, because parties' expected utility incorporates something more than mere seat gains: the increase in the *probability* of forming a single-party government, for the parties favoring the reform. In the presence of electoral uncertainty this is utility enhancing for both, in expected terms.

<sup>6</sup>Fernández-de-Córdoba and Penadés (2009) also study the choice of electoral institutions (from a broad set of rules) by parties that try to maximize the minimum number of seats for a given share of votes. Their analysis shows that the maximin choice is the D'Hondt rule, generally equivalent to the Droop formula. The scope of our analysis is different as we focus on dominant parties' incentives for strategic coordination, driven by their desire to form a single-party government.

<sup>7</sup>This type of proportionality distortion is an ordinary one as it gives a permanent advantage to the big parties (the largest of which will always receive the premium). Hence, we also use the term *plurality premium* in order to stress the fact that the simple plurality winner will always be entitled to the premium. Greece applies this electoral rule since the 2007 elections. Another example of how this mechanism is applied are the 2006, 2008 and 2013 Italian legislative elections.

allows our model to capture not only major (of which historically there have been a few) but also minor electoral reforms which appear to be more frequent (Katz 2005).<sup>8</sup> The reason that we have chosen to model distortions of the PR rule using the plurality premium technology instead of introducing exclusion quotas (or altering the district magnitude) is that these technologies tend to benefit (in terms of seats) *both* dominant parties, since they will surely be represented in parliament. Hence, their incentives to enact a majoritarian reform are easily aligned (perhaps this is the reason that exclusion quotas are in place in most countries). We, on the other hand, are interested in exploring whether both dominant parties can actually agree to implement a majoritarian reform, such as the premium system, that only benefits the winner. Nevertheless, the way we model electoral reform is robust to alternative interpretations and can also account for the effects of small party entry on electoral rule choice.

## 2 The Model

### 2.1 Set-up and Preferences

Formally, we let  $\mathcal{N} = \{1, 2, 3\}$  to be the set of parties who will participate in forthcoming elections. All parties in  $\mathcal{N}$  are assumed to be represented in the current parliament. Each party  $i \in \mathcal{N}$  holds a proportion of seats  $s_i^0$  in the current parliament, such that  $\sum_{i=1}^n s_i^0 = 1$ . Moreover, parties have information about each other's expected vote share in the forthcoming elections. Party 3, is assumed to be a *non-winning* party. That is, its vote share shall never exceed that of party 1 or party 2. The expectations on future vote shares are formed by information that is *commonly* available to *all* parties. The vote share of the non-winning third party is assumed to be fixed at a level  $v_3$ , whereas, the vote shares of the other two parties are subject to uncertainty.<sup>9</sup> Formally, the vote share of party 1 in the coming elections will be modeled as a random variable:

$$v_1 \sim \Phi, \text{ where } \Phi \text{ is a uniform distribution in } [a_1, b_1] \subset [0, 1].^{10}$$

It follows that, since  $v_2 = 1 - v_1 - v_3$ , it must be the case that  $v_2 \in [a_2, b_2]$  where  $a_2 = 1 - b_1 - v_3$  and  $b_2 = 1 - a_1 - v_3$ . These vote shares need to satisfy  $\sum_{i=1}^n v_i = 1$  and  $v_3 < a_i$ , for all  $i \in \{1, 2\}$ . That is, both  $b_1$  and  $b_2$  are smaller than  $1 - 2v_3$  and  $v_3 < 1/3$ . We assume that  $\frac{1-v_3}{2} \in (a_1, b_1)$  which means that each of the two dominant parties has a positive probability of being the winner of the coming elections.

<sup>8</sup>By minor electoral reforms we mean small changes in the magnitude or in the boundaries of certain districts.

<sup>9</sup>Practically, our model can be viewed as a multi-party model if one interprets  $v_3$  as the sum of the vote shares of all other parties. As long as  $v_3 < \min\{v_1, v_2\}$  our third party stands for the collection of all other small parties.

<sup>10</sup>The main results of this paper are also robust when one considers a more general class of distributions  $\Phi(\cdot)$ , as we explicitly show in the proof of Lemma 1 (Appendix A).



As it will be evident in the analysis section, the way we chose to model uncertainty is a reduced form of assuming that the vote share distribution  $\{v_1, v_2, v_3\}$  is a random draw from a multivariate distribution  $Z(v_1, v_2, v_3)$  under the restriction that  $v_3 < \min\{v_1, v_2\}$ . That is, certainty about  $v_3$  is not required for our results but improves significantly the readability of the formal parts of the paper.

The share of seats that party  $i \in \mathcal{N}$  occupies in the new parliament will be defined as  $s_i^l(v_i)$ , where  $l$  will be the applied electoral rule. As stated before, we do not limit our attention to a single electoral rule. Rather, we want to consider a wide range of electoral rules. In order to capture the big diversity of electoral rules existing in the world we will adopt the following mechanism  $l$  that is based on a variation of the PR rule with the use of a seat premium allocated to the first party. That is,  $l$  is the proportion of parliamentary seats allocated to the winning party as a premium (plurality premium). Depending on the degree of the introduced distortion,  $l \in [0, 1]$ , the share of seats of party  $i \in \{1, 2, 3\}$  in the new parliament will be:

$$s_i^l(v_i) = \begin{cases} (1-l)v_i & , \text{ if } v_i < \max\{v_j\}, \text{ for } j \neq i \\ (1-l)v_i + \frac{l}{2} & , \text{ if } v_i = \max\{v_j\}, \text{ for } j \neq i \\ (1-l)v_i + l & , \text{ if } v_i > \max\{v_j\}, \text{ for } j \neq i \end{cases}$$

That is, the premium  $l$  is awarded to the (simple plurality) winner of the election.<sup>11</sup> Given our assumptions that  $v_3 < 1/3$  we have that  $v_3 < \min\{v_1, v_2\} < \max\{v_1, v_2\}$ . That is, the third party is *never entitled* to this bonus as a result of never winning elections. In fact, much of those bonus seats are taken from it. The above seat allocation mechanism allows us to capture a wide variety of electoral rules, from pure PR to mixed systems and FPTP.<sup>12</sup> The first part is the proportional allocation of the seats minus the reserved premium, whereas the second part is the bonus given to the winner. The utility of a party  $i \in \mathcal{N}$  will be defined as:

$$u_i(v_1, v_2, v_3, l) = \begin{cases} s_i^l(v_i) & , \text{ if } s_i^l(v_i) \leq 1/2 \\ 1 & , \text{ if } s_i^l(v_i) > 1/2 \end{cases}$$

This formulation of utility captures the desire that parties have for single-party governments and the smaller marginal utility of an extra parliamentary seat once a party has gained more than one half of the seats. Yet, it is open to two main criticisms: Firstly, it is such that the marginal utility of an additional

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<sup>11</sup>Note that in our model (with three parties) the threshold of awarding the bonus to the winner is  $(1 - v_3)/2$  simply because this is the threshold for *plurality*.

<sup>12</sup>To see this, consider the case of  $l = 0$ . Then, our electoral rule is pure PR, whereas in the case of  $l = 1$ , it transforms into a FPTP system where the whole country is one district.

seat for a party that already has a majority in the parliament is zero; and secondly, it suggests that when a party cannot form a single-party government then its utility solely depends on its seat share independently of whether it participates in a coalition government or if it stays in opposition.

In the Appendix we show that more general assumptions regarding office rents (where parties can receive rents, albeit smaller, if they participate in a coalition government) and the marginal utility of an extra seat for a party has the majority in the parliament (which is again less than the marginal utility when a party is in a coalition government or in opposition, but need not necessarily be zero) do not contradict any of the forces that are crucial for our results. In fact, we show that those results are equivalent with the simpler case that we exhaustively analyze in the paper.<sup>13</sup> That is, our simplified preference structure is a rough but fair approximation of more general preference structures.

## 2.2 The Game Structure

After defining the preferences of the parties, we proceed with the structure of the electoral reform game. Formally, the game has three stages:

(i) The current rule is  $l = 0$  (pure PR) and the party with the largest share of seats in the current parliament shall bear the role of the *proposer* of an electoral reform (without loss of generality we assume that the proposer is always party 1). That is, it will propose  $l \in [0, 1]$ .

(ii) Parties vote on the proposal  $l$ . If the votes in favor of the reform surpass a given threshold  $W \in [0, 1]$  (given exogenously by the constitution) then, the electoral reform proposal is accepted. In the opposite case, if the proposal does not gather sufficient parliamentary support  $W$ , the electoral reform is cancelled. That is, defining as  $l^*$  the rule according to which forthcoming elections will take place, we note that  $l^*$  can be either the proposed  $l$  or 0 (which means that proposal  $l$  was defeated and the PR was maintained).

(iii) Elections take place and each party obtains its new seat share in the parliament and computes its utility according to its vote share and the applied electoral rule  $l^*$ .

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<sup>13</sup>The formal proofs (Appendix A) and the analysis of the general case (Appendix B) will demonstrate that it is not the constant nature of  $u_i(\cdot)$  for  $s_i^l(v_i) > 1/2$  that drives our results, but the assumption that the gain from an extra seat should be less for a party that already has a majority in the parliament compared to that when this party is in a coalition government or in opposition.

### 3 Results

Since we have assumed that  $v_3$  is fixed and common knowledge, there are two important thresholds as far as the vote share of a dominant party is concerned. The first one,  $(1 - v_3)/2$ , defines the necessary vote share for a party to be the plurality winner and get the bonus,  $l$ . The second,  $\frac{1/2-l}{1-l}$ , is the vote share that the first party needs in order to get the majority of seats in the parliament and form a single-party government given electoral rule  $l$ . Obviously, if  $(1 - v_3)/2 \geq \frac{1/2-l}{1-l}$  the plurality winner will automatically have a majority in the parliament. We note here that the critical value of the bonus,  $l$ , at which these thresholds become identical is  $\frac{v_3}{1+v_3}$ ; that is  $(1 - v_3)/2 = \frac{1/2-l}{1-l} \Leftrightarrow l = \frac{v_3}{1+v_3}$ . Hence, for both  $i \in \{1, 2\}$ , when  $l < \frac{v_3}{1+v_3}$  we have that:

$$Eu_i(l) = \begin{cases} \frac{1}{(b_i - a_i)} \left[ \int_{a_i}^{(1-v_3)/2} v_i(1-l)dv_i + \int_{(1-v_3)/2}^{\frac{1/2-l}{1-l}} [v_i(1-l) + l]dv_i + \int_{\frac{1/2-l}{1-l}}^{b_i} dv_i \right] & \text{if } b_i > \frac{1/2-l}{1-l} \\ \frac{1}{(b_i - a_i)} \left[ \int_{a_i}^{(1-v_3)/2} v_i(1-l)dv_i + \int_{(1-v_3)/2}^{b_i} [v_i(1-l) + l]dv_i \right] & \text{if } b_i \leq \frac{1/2-l}{1-l} \end{cases}$$

and when  $l \geq \frac{v_3}{1+v_3}$  we have that:

$$Eu_i(l) = \frac{1}{(b_i - a_i)} \left[ \int_{a_i}^{(1-v_3)/2} v_i(1-l)dv_i + \int_{(1-v_3)/2}^{b_i} dv_i \right].$$

Bearing all these in mind we first describe properties of these expected utility functions for the two big parties in order to understand the nature of the choice problem that they face.

**Lemma 1** *For both  $i \in \{1, 2\}$  and every admissible  $v_3$  the expected utility function  $Eu_i(l)$  is continuous on  $[0, 1]$ , convex on  $[0, \frac{v_3}{1+v_3}]$ , strictly convex on a subset of  $[0, \frac{v_3}{1+v_3}]$  and decreasing on  $(\frac{v_3}{1+v_3}, 1]$ .<sup>14</sup>*

Lemma 1 implies that there are *exactly two* values of  $l$  that are candidates for maximum: either 0 or  $\frac{v_3}{1+v_3}$ . The trade-off that a dominant party faces is the following: a larger  $l$  increases the probability that this party forms a single-party government conditional on it being the plurality winner while it decreases its parliamentary seat share conditional on it not being the plurality winner. The reason why  $Eu_i(l)$  is decreasing in  $(\frac{v_3}{1+v_3}, 1]$  for both  $i \in \{1, 2\}$  is because any  $l$  larger than  $\frac{v_3}{1+v_3}$  is such that the plurality winner has a majority in the parliament and hence forms a single-party government with probability one - a dominant party is essentially indifferent between  $l_1$  and  $l_2$ , where  $\frac{v_3}{1+v_3} < l_1 < l_2$ , conditional on it being the plurality winner but it strictly prefers  $l_1$  to  $l_2$  conditional on it not being the plurality winner.

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<sup>14</sup>All proofs are in Appendix A.

The convexity between 0 and  $\frac{v_3}{1+v_3}$  is due to the strong desire for forming a single-party government (see Figures 1 and 2).

[Insert Figures 1 and 2 about here]

For expositional ease, we define function  $f_i(a_i, b_i, v_3) \equiv Eu_i(l = \frac{v_3}{1+v_3}) - Eu_i(l = 0)$ , which measures the expected utility gains (or losses) for a party  $i \in \{1, 2\}$  when there is a change in the electoral rule from 0 to  $\frac{v_3}{1+v_3}$ . For  $b_i < \frac{1}{2}$ , simple algebra yields that:

$$f_i(a_i, b_i, v_3) = \frac{1}{(b_i - a_i)} \left[ (b_i - \frac{1-v_3}{2})(2 - b_i - \frac{1-v_3}{2}) - (\frac{v_3}{1+v_3})(\frac{1-v_3}{2} + a_i)(\frac{1-v_3}{2} - a_i) \right]$$

Otherwise, for  $b_i \geq \frac{1}{2}$ , we have that:

$$f_i(a_i, b_i, v_3) = \frac{1}{(b_i - a_i)} \left[ \frac{v_3}{2} + \frac{(v_3)^2}{4} + \frac{1}{4} \left( \frac{2v_3^2 - v_3 - v_3^3 + 4v_3 a_i^2}{v_3 + 1} \right) \right].$$

Intuitively,  $f_i(\cdot) \geq 0$ , means that Party  $i$  has good chances of winning the elections (to see this notice that  $f_i(\cdot)$  is strictly positive, for  $b_i \geq \frac{1}{2}$ ) and hence, it prefers  $l = \frac{v_3}{1+v_3}$ .<sup>15</sup> That is,  $f_i(\cdot)$  measures the difference between the *expected gain* when  $l = \frac{v_3}{1+v_3}$  (instead of  $l = 0$ ) is applied and as a result, the winner is able to form a single-party government and the *expected loss* from running second when  $l = \frac{v_3}{1+v_3}$  is applied instead of  $l = 0$ .

The nature of the parties' expected utilities has an important implication as far as a modelling assumption is concerned: it renders the structure of the bargaining process irrelevant. Given that we have adopted a "take-it-or-leave-it" bargaining protocol, one might rightfully worry that our results will be driven by this particular assumption that does not allow for counter-proposals. Whereas, this statement would have been *generically* true in any other context, in this particular set up, enriching the bargaining process plays absolutely no role: a) any reasonable mechanism should result to a majoritarian reform (from 0 to  $\frac{v_3}{1+v_3}$ ) when the current seat share of the parties whose expected utility is maximized at  $\frac{v_3}{1+v_3}$  is larger than  $W$  and b) any reasonable mechanism should result to no reform when the current seat share of the parties whose expected utility is maximized at  $\frac{v_3}{1+v_3}$  is smaller than  $W$ .

Since the electoral rule determines the seat allocation for the parties and thus, their utility, the proposer - party 1 - will propose an electoral rule  $l \in [0, 1]$  such that, given the threshold  $W \in [0, 1]$ , maximizes its expected utility. If  $W \leq s_1^0$  party 1 will propose  $l \in [0, 1]$  such that  $l \in \arg \max \{Eu_1(l)\}$ . In other words, if the current seat share of the proposer exceeds the required threshold for enacting an electoral reform, the proposer faces an unconstrained maximization program.

<sup>15</sup>Notice that  $f_i(\cdot)$  is strictly increasing in  $a_i$  and replace  $a_i$  with  $v_3$  in the equation (since by assumption, we have  $v_3 < a_i$ ). Then, observe that for every  $v_3 > 0$  it is always true that  $f_i(\cdot) > 0$ .

**Proposition 1** *Assume the electoral reform process is trivial ( $W \leq s_1^0$ ). Then,  $l^* = \frac{v_3}{1+v_3}$  if and only if  $f_1(a_1, b_1, v_3) \geq 0$ , otherwise  $l^* = 0$ .*

On the contrary, if  $W > s_1^0$  the proposer needs the support of at least one of the two other parties in order to implement an electoral rule reform. Lemma 2 demonstrates the behavior of the third party in such cases.

**Lemma 2** *The third party always prefers the PR rule ( $l = 0$ ) to any other rule.*

This lemma suggests that an electoral reform is possible only if  $W \leq 1 - s_3^0$  - this is what we will be assuming for the rest of the paper. Hence, if  $W > s_1^0$  party 1 is required to secure party 2's consent in order for an electoral rule change to take place. Party 1 in this case faces a constrained maximization problem and it has to propose:

$$l \in \arg \max \{Eu_1(l) \mid \text{s.t. } Eu_2(l) \geq Eu_2(0)\}$$

But is it possible that this maximization problem has an interior solution? That is, can a majoritarian electoral rule be adopted when the consent of the other dominant party is required? Lemma 1 suggests two things: a) the solution of the above constrained maximization problem is either 0 or  $\frac{v_3}{1+v_3}$  and b)  $\frac{v_3}{1+v_3}$  is a solution of the above maximization problem if both  $f_1(a_i, b_i, v_3) \geq 0$  and  $f_2(a_i, b_i, v_3) \geq 0$ .

When these conditions are satisfied, party 1 with the necessary consent of party 2, sponsors a majoritarian electoral rule reform ( $l = \frac{v_3}{1+v_3}$ ). But is it even possible that these conditions are simultaneously satisfied? That is can it be that both dominant parties consent to a majoritarian reform? It is natural to expect that the expected winner wants such a reform. But could it be the case that the expected loser also consents to it? The answer is yes, even in very asymmetric cases. Dominant parties have very strong incentives to collude and pass a majoritarian reform. The reason is that introduction of a seat premium increases the *probability* of forming a single-party government in the event of an electoral victory. This is appealing not only for the party which is expected to win but for the party that is *less* likely to win as well (as long as its chances of winning are not too small). We provide a simple illustration below.

**Example 1** *Let  $v_1 \sim U[0.44, 0.50]$ ,  $v_2 \sim U[0.40, 0.46]$  and  $v_3 = 0.10$ . Then, both parties prefer  $l = \frac{v_3}{1+v_3}$  to the PR rule.*

In this example party 1 has a 5/6 chance of winning elections. Party 1 is the expected winner and prefers  $l = \frac{v_3}{1+v_3}$  to the PR ( $l = 0$ ). What is easily checked is the fact that party 2 - the expected loser -

also prefers the majoritarian rule. To see this replace  $a_2 = 0.40$  and  $b_2 = 0.46$  and observe that condition  $(b_2 - \frac{1-v_3}{2})(2 - b_2 - \frac{1-v_3}{2}) > (\frac{v_3}{1+v_3})(\frac{1-v_3}{2} + a_2)(\frac{1-v_3}{2} - a_2)$  is always satisfied with strict inequality, implying  $Eu_2(l = \frac{v_3}{1+v_3}) > Eu_2(l = 0)$ . As a result, even with this high asymmetry over the distribution of votes, it is still the case that party 2 strictly prefers the majoritarian rule  $\frac{v_3}{1+v_3}$ . Next, we formalize the idea that equal chances of victory are enough for a collusion among the dominant parties to take place.

**Lemma 3** *In the symmetric case ( $a_1 = a_2$ ) both dominant parties always strictly prefer  $l = \frac{v_3}{1+v_3}$  to  $l = 0$ .*

The proposition that follows is actually a corollary of the above lemma. Since in the symmetric case  $f_i(\cdot) > 0$  for both parties and the inequality is strict, we can show that in asymmetric cases in a neighborhood around the symmetric one the inequality still holds. Hence, the two dominant parties should still have incentives to collude and implement a majoritarian reform.

**Proposition 2** *For every admissible  $a_1$  there exists  $z > 0$  such that both dominant parties strictly prefer  $l = \frac{v_3}{1+v_3}$  to  $l = 0$  if  $a_1 - a_2 \in (-z, z)$ .*

Proposition 3 formalizes the main finding of the paper. In an environment of electoral uncertainty and when the electoral reform is *non-trivial*, both dominant parties desire to implement the same majoritarian reform conditional on both having sufficiently similar chances of winning the elections (as we saw in the example above these sufficiently similar chances may in fact be very different). Hence this model helps us understand not only major electoral reforms from PR to majoritarian systems but minor electoral reforms too. If the vote share of the third party ( $v_3$ ) varies slightly between elections but electoral uncertainty is high then, this model would predict that we should observe many minor changes in the disproportionality of the electoral rule taking place.<sup>16</sup>

### 3.1 Testable Implications

An important implication of equilibrium *uniqueness* is that the value of the seat premium  $l = \frac{v_3}{1+v_3}$  is such that it always allows the winner to form a single-party government. That is, if the perks of office are high enough and if there is sufficient electoral uncertainty, office motivated parties can agree to distort the PR rule in order to consolidate the two-party system. Moreover, a simple comparative statics analysis on the

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<sup>16</sup>This model could further be used to study transitions from less ( $l = \frac{v_3}{1+v_3}$ ) to more proportional ( $l = 0$ ) rules too. Since, there are only two candidate values of  $l$  for an optimum, in equilibrium it must be either  $l = 0$  or  $l = \frac{v_3}{1+v_3}$ . If the proposer has enough seats to implement the reform single-handedly, which is usually the case when the a majoritarian rule is the status-quo rule, the proposer will chooses the PR rule ( $l = 0$ ) if it expects to lose the election with sufficiently high probability. Otherwise, it picks  $l = \frac{v_3}{1+v_3}$ , which allows it to form a single-party government in the event of an electoral victory.

equilibrium value of the seat premium,  $l(v_3) \equiv \frac{v_3}{1+v_3}$ , reveals that proportionality distortion is increasing in the electoral power of smaller parties (captured by  $v_3$  in our model). That is,  $\partial(\frac{v_3}{1+v_3})/\partial v_3 > 0$ . This prediction offers an additional insight in the relationship between party-system fragmentation, new-party entry and electoral rule reform. In essence,  $v_3$  is a proxy for the fragmentation of the party-system as the *ENP* index is *positively* and *monotonically* related and  $v_3$ , as long as  $v_3 < 1/3$ .

First, let us understand by  $v_3$  the sum of vote shares of all other non-winning parties (assume there are  $n$  such parties), except for the two most voted parties. Moreover, let us assume that the two most voted parties parities -the dominant ones- are symmetric (they receive equal vote shares in expectation) and that moreover the  $n$  smaller parties are symmetric as well (each of receives a vote share equal to  $v$ ). Then, the vote share of a dominant party is  $\frac{(1-nv)}{2} > v$ . Since  $nv = v_3$ , it follows that  $\frac{(1-nv)}{2} > v$  is equivalent to  $v_3 < \frac{n}{2+n}$  which is unambiguously true (that is, it is true for any  $n$ ) if  $v_3 < 1/3$ . In party-systems which satisfy these conditions (two dominant parties and  $v_3 < 1/3$ ) our theory predicts that  $l = \frac{v_3}{1+v_3}$ ; in all other cases it may be the case that  $v_3$  has any other kind of effects on the electoral rule or it may not have any effect at all.

Especially for the relevant range of the values of  $v_3 < 1/3$  and under the assumption presented above (two big parties and  $n$  smaller ones) one can compute the *Effective Number of Parties* (*ENP*) index as a function of  $v_3$  and  $n$  and show that it is *strictly* and *monotonically* increasing in  $v_3$ , for all  $v_3 < 1/3$ . Formally, we have:

$$ENP = \frac{1}{\sum_i (v_i)^2} = \frac{1}{2\left(\frac{1-nv}{2}\right)^2 + nv^2} = \frac{1}{2\left(\frac{1-v_3}{2}\right)^2 + \frac{(v_3)^2}{n}}.$$

It is then easily verified that  $\frac{\partial ENP}{\partial v_3} > 0$  and  $\frac{\partial ENP}{\partial n} > 0$  if  $v_3 < 1/3$ .<sup>17</sup> This implies that we can substitute  $v_3$  with the *ENP* index in our empirical estimation. Hence, our model's prediction that for  $v_3 < 1/3$  an increase in  $v_3$  should lead to a less proportional electoral rule (an increase in the size of premium  $l$ ) can be stated in terms of the *ENP* index which measures party-system fragmentation.

**Hypothesis 1** *If  $v_3 < 1/3$ , an increasing ENP leads to a less proportional rule.*

Our theory does not offer conclusive implications for the cases in which  $v_3 > 1/3$ , which are undoubtedly many. This is why, its resulting hypothesis on the relationship between party-system fragmentation and electoral rule disproportionality is a *conditional* one (stated only for  $v_3 < 1/3$ ). Hence, it does not necessarily contradict Colomer's *unconditional* hypothesis (2005) which states that an increase in the

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<sup>17</sup>That is, we have  $\frac{\partial l(v_3)}{\partial v_3} = \frac{\partial l(v_3)}{\partial ENP} \times \frac{\partial ENP}{\partial v_3} > 0$ , implying  $\frac{\partial l(v_3)}{\partial ENP} > 0$ .

fragmentation of the party-system (an increase in the *ENP* index) leads to more proportional rules. In fact, our theory provides a rational choice explanation to the findings by Riera (2013): as long as the party-system is characterized by the presence of two dominant parties (that is,  $v_3 < 1/3$ ) our model predicts that an increase in the vote share of smaller (non-winning) parties causes the adoption of less, not more, proportional rules.

In the section that follows, we test this *conditional* hypothesis together with our main comparative statics prediction on the size of the seat premium ( $l$ ). In the Appendix, we will also present a series of case-studies on recent majoritarian electoral rule reforms in order to examine more closely our equilibrium mechanism (strategic coordination between dominant parties).

## 4 Empirical Analysis

In order to test empirically the predictions of our model we collect electoral and institutional data for 23 OECD countries for the period from 1960 to 2006 from two different sources: the Comparative Political Data Set I (CPDS I) and the data archive used by Carey and Hix (2011).<sup>18</sup> In total we have observations from more than 300 elections for a period of almost fifty years. To operationalize the measurement of our dependent variable, the disproportionality (majority bias) of the electoral rule, we use the *Losemore-Hanby* (*LH*) index of disproportionality which measures the aggregate absolute deviations between seats and votes fractions and is defined as follows:

$$LH_{n,t} = \frac{1}{2} \sum_i \left( |v_{i,n,t} - s_{i,n,t}^l| \right),$$

where  $v_{i,n,t}$  is the vote share and  $s_{i,n,t}^l$  is the seat share of party  $i$  in country  $n$  in election (year)  $t$  when rule  $l$  is applied. The use of the *Losemore-Hanby* over the *Gallagher* index of disproportionality, which is more commonly used in the literature, is due to the fact that the *LH* index fits our theory much better, for it is indifferent to fragmentation among smaller parties and is consistent with our interpretation of  $v_3$  as the aggregate vote share of small parties.<sup>19</sup> Our model suggests that based on the degree of disproportionality (measured by the *Losemore-Hanby* index) we should expect the distribution of applied electoral rules to be concentrated around two clusters: when the index is equal to 0 (when there is no

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<sup>18</sup>Detailed variables definitions and the codes for replicating our estimation results can be found in Appendix C.

<sup>19</sup>The squaring of deviations in the Gallagher index is intended to make it different when the “third party” is a single party, as opposed to the situation (as in our model) where it is the aggregate of small parties. Hence, the Gallagher index introduces a difference where the model (and the *Losemore-Hanby* index) prefer to see equivalent situations. Nevertheless, our findings are robust to using the Gallagher index and additional results are available by the authors upon request.



reform and  $l = 0$  is applied), and when the index is around .095 (when  $l = \frac{v_3}{1+v_3}$  is applied).<sup>20</sup> Preliminary analysis of aggregate data from our sample seems to confirm this prediction (Fig. 3), despite the rough approximation technique that we employ.

But, in addition to providing aggregate empirical evidence, we also estimate the full version of our model. Our two key explanatory variables are: the *Effective Number of Parties* (*ENP*) index, defined as  $1/\sum_i (v_{n,t}^i)^2$ , which measures the fragmentation of the party-system and the dispersion of electoral power among parties, and the sum of vote shares of all non-dominant parties (what we call  $v_3$ ). Moreover, to increase the fit between our model and the empirical estimation we also introduce two control variables in order to account for the competitiveness of the electoral race (degree of electoral uncertainty) and the ability of parties to extract rents from holding office: the *margin of victory* of the winning party and a dummy variable indicating *high institutional constraints*, respectively. Formally, we estimate the following equation (Model 1):<sup>21</sup>

$$LH_{n,t} = \beta_0 + \beta_1 ENP_{n,t} + \mathbf{X}'_{n,t} * \boldsymbol{\gamma} + \alpha_n + \lambda_t + \varepsilon_{n,t} \quad (1)$$

where  $ENP_{n,t}$  is the *Effective Number of Parties* index computed for country  $n$  in election  $t$ ,  $\alpha_n$  and  $\lambda_t$  are unit-specific(country dummies) and time-specific (year dummies) fixed effects and  $\mathbf{X}'_{n,t}$  is the set of other control variables (margin of victory, constraints dummy, a dummy variable indicating frequency of coalition governments, the number of parties participating in government, a dummy indicating whether the regime is a presidential or a parliamentary democracy and the number of years that a country is an established democracy).<sup>22</sup> Given our model's predictions and our conjecture we expect  $\beta_1 > 0$  if the sum of vote shares of the remaining non-dominant parties is less than a third (recall that  $\frac{\partial ENP}{\partial v_3} > 0$ , which also implies  $\frac{\partial l(v_3)}{\partial ENP} > 0$ , for  $v_3 < 1/3$ ).

[Insert Table 2 and Figure 3 about here]

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<sup>20</sup> Assuming symmetry, if one replaces the vote shares of dominant parties with their expected ones ( $E[v_i] = \frac{1-v_3}{2}$ ) and computes their seat shares  $s_i^l(v_i)$  as a function of the applied rule  $l$  (by Lemma 3 in the symmetric case we have  $l = \frac{v_3}{1+v_3}$ ), after some algebra one can get the predicted *Losmore-Hanby* index for country  $n$  at election  $t$  which becomes:  $\widehat{LH}_{n,t} \left( \frac{v_3}{1+v_3} \right) = \frac{(v_3)_{n,t}}{2}$ . Since in our sample, the average vote share of third parties (other than the two major ones) is 0.19, one can calculate the *LH* index for the symmetric case and obtain a value of 0.095.

<sup>21</sup> While most of the literature uses the *ENP* index our theoretical prediction is stated in terms of  $v_3$ . One problem with the *ENP* index is that it is not invariant to how  $v_3$  is distributed across different small parties. Hence, in order to address this issue and increase the fit between our theoretical and empirical predictions, we estimate again Model 1 by replacing *ENP* with  $v_3$ . This is done when we estimate Model 3 (Appendix B).

<sup>22</sup> The rationale for using those additional controls is similar to that of using the *margin of victory* and the *high institutional constraints* dummy in order to control for electoral uncertainty, the size and the allocation of office perks among different parties and branches of the executive (which implicitly determines the value of a single-party government). Similarly, the age of the democratic regime is a proxy for a more established (and perhaps more inclusive) democracy.

First we estimate Model 1 using the full sample. Then, we restrict our sample to contain only those cases where  $v_3 < 1/3$ , as our theory dictates. The results of estimating (1) in both samples (results for the full sample in odd columns) are presented in Table 2. In columns 1 and 2 we estimate a simpler version of Model 1 without the inclusion of fixed effects, while in columns 3 to 6 we estimate the fixed effects (FE) model. The results support our hypothesis, as in the restricted sample the coefficient on  $ENP$  is always positive and statistically significant at any conventional level. They can be interpreted as follows: an increase in the  $ENP$  by one standard deviation is associated with a more than 1.5 standard deviation increase in the disproportionality of the electoral rule, which implies that our results are also qualitatively significant. Moreover, when we compare those results with the ones obtained when the model is estimated using the full sample, we observe that  $\beta_1$  either fails to be statistically different from zero (column 1) or is a smaller in magnitude positive number. The latter follows from the fact that while the effect of  $ENP$  on electoral rule disproportionality is unambiguously positive when  $v_3 < 1/3$ , whenever  $v_3 > 1/3$  our model does not offer a conclusive prediction (it can also be negative). Hence, one can expect that the coefficient  $\beta_1$  will drop in magnitude when the sample includes those observations.

Furthermore, in order to explore in more detail this conditional relationship between the  $ENP$  index and the disproportionality of the electoral rule we estimate a variation of the previous model, where instead of restricting the sample to those cases where  $v_3 < 1/3$ , we interact the  $ENP$  variable with  $v_3$  (the variable that measures the sum of vote shares of all non-dominant parties). That is, we formally estimate the following equation (Model 2):

$$LH_{n,t} = \beta_0 + \beta_1 ENP_{n,t} + \beta_2 ENP_{n,t} * (v_3)_{n,t} + \mathbf{X}'_{n,t} * \boldsymbol{\gamma} + \alpha_n + \lambda_t + \varepsilon_{n,t} \quad (2)$$

where  $(v_3)_{n,t}$  is the sum of vote shares of all non-dominant parties in country  $n$  in election (year)  $t$  and all other variables are as defined above. Again, we expect  $\beta_1 > 0$  but this time we also expect  $\beta_2 < 0$ . That is, the positive effect of  $ENP$  on the disproportionality of the electoral rule is mitigated as  $v_3$  is increasing. We present the results of estimating equation (2) in Table 3.<sup>23</sup>

Once more, the results we get are consistent with our model's prediction. In all specifications the coefficients on  $ENP$  ( $\beta_1$ ) and the interaction term ( $\beta_2$ ) have the expected signs and are statistically significant at the conventional level. We also get an analogous picture, when we look at the estimates of

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<sup>23</sup>The expectation that  $\beta_2 < 0$  is also a direct implication of our theoretical model. Recall that we have shown  $\partial ENP / \partial v_3 > 0$  which implies that  $\beta_2 = \frac{\partial^2 LH}{(\partial ENP)(\partial v_3)}$  should have the same sign with  $\frac{\partial^2 l(v_3)}{(\partial v_3)^2} = -\frac{2(1+v_3)}{(1+v_3)^4} < 0$ . Moreover, since  $ENP$  and  $v_3$  are monotonically related, we also estimate Model 2.b where we replace  $ENP$  with  $v_3$ . In this case it is straightforward to check that  $\beta_2 = \frac{\partial^2 LH}{(\partial v_3)^2}$  should have the exact same sign with  $\frac{\partial^2 l(v_3)}{(\partial v_3)^2} < 0$ . We present those estimation results in columns 6 and 7 (Table 3).

Model 2.b. That is, the initial effect of *ENP* on disproportionality is positive (for low values of  $v_3$ ) and gradually decreasing as the electoral power of the non-dominant parties is rising. We illustrate this point in greater clarity in figures 4 and 5, where we plot the conditional (Fig. 4) and average (Fig. 5) marginal effects of *ENP* and  $v_3$  respectively, on the *LH* index of disproportionality for a given level of  $v_3$ . In both cases, observe that for all  $v_3 < 1/3$  the *marginal effect* of an increase in the ENP (or  $v_3$ ) is unambiguously *positive* and always *statistically different from zero* at the conventional 5% level, but is decreasing in  $v_3$  as expected (recall that the slope has the same sign with  $\beta_2 < 0$ ). Moreover, notice that for values of  $v_3 > 1/3$  the effect is not statistically distinguishable from zero at the 5% level (that is, it can also be negative). Thus, the results of both models provide additional evidence in support of our comparative statics prediction and our conditional hypothesis.<sup>24</sup>

[Insert Table 3 and Figures 4, 5, 6 and 7 about here]

In sum, our empirical findings yield adequate support to the comparative statics prediction of our model. As our model suggests and our empirical findings seem to verify, in any given political environment, there exist an optimal (from the perspective of dominant parties) electoral rule, which is chosen strategically by them as a response to changes in the degree of political competition in order to maintain their dominant position. If that is the case, in party-systems with two major parties, we should observe some convergence between the actual and the predicted degree of electoral rule disproportionality. In fact, we do observe such a convergence (Fig. 6) when we plot the actual *Losemore-Hanby* index of disproportionality against the predicted one (recall that in the symmetric case  $\widehat{LH}_{n,t} = \frac{(v_3)_{n,t}}{2}$ ). Moreover, in addition to presenting only aggregate measures of fitness, we also estimate the complete model and present the results in Appendix B. In figure 7 we present a visual summary of those results.

## 5 Final Remarks

Our analysis has so far demonstrated that increased political competition (e.g., a rise in electoral uncertainty or new-party entry) may not always lead to more political representation (via more permissive electoral rules). Yet, there might be an additional factor that can influence the choice of electoral rules, ideology. Until now, the literature on electoral rule reform has solely focused on *seat-maximizing* parties,

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<sup>24</sup>Note that the statement regarding the threshold is not biconditional. That is, while  $v_3 < 1/3$  always implies that the two dominant parties are never threatened by any of the other parties the converse statement is not necessarily true as this depends on how  $v_3$  is distributed among the other parties. That is why we get a clear positive (and decreasing) effect when  $v_3 < 1/3$ , but we do not get a clear prediction when  $v_3 > 1/3$ .

omitting completely from the discussion this additional dimension.<sup>25</sup> Moreover, the static nature of these models (including ours) raises some questions regarding the dynamic stability of the equilibrium electoral rules. Although we do not aspire to conduct an exhaustive analysis on the effect of ideology in the choice of electoral rules, we briefly discuss below how endowing parties with well-defined preferences over ideology (in a left-right dimension) can under certain conditions guarantee the dynamic stability of electoral rules, thus offering some useful insights on the role of ideology in the design of electoral institutions.

If parties know that the electoral rule that they will design will be relatively long-lived, then the uncertainty that we assume about the next elections can be easily interpreted as the distribution of the expected outcomes for the many elections that will take place under a certain electoral rule. Moreover, in this dynamic context, the symmetric scenario (the scenario in which both parties are expected to win the next election with equal probability) can be interpreted as if each one of the two major parties is expecting to win half of the times during the period in which this electoral rule will be applied. For instance, in most of the countries in our sample, each of the two largest parties won approximately half of the elections in any arbitrary long period. In such a framework, incentives for strategic coordination over adopting a more majoritarian rule may conflict with ideological differences between the two dominant parties. For example, if at the time of the electoral rule choice the ideologies of the two major parties diverge, then it seems more plausible that they will choose PR over a more restrictive (majoritarian) rule. The intuition is that, by sticking to a purely proportional rule, dominant parties can utilize the PR as an insurance device against the risk of having of a single-party government being formed by a polar-opposite party. That is, extreme ideological divergence can potentially cancel off any incentives for strategic coordination which we have identified. But, in turn, when parties compete under a PR rule they tend to adopt highly polarized political platforms (Cox 1990; Calvo and Hellwig 2011).<sup>26</sup>

On the other hand, if at the time of electoral rule choice the two major parties exhibit ideological proximity (as we have implicitly assumed in this paper), then they choose a majoritarian electoral rule, and when parties compete under majoritarian electoral rules they tend to adopt more convergent platforms (this holds especially for major parties; see Calvo and Hellwig 2011). So, we observe that two dynamically stable paths arise: PR leads to platform polarization and platform polarization leads to PR while majoritarian systems lead to platform convergence and platform convergence leads to majoritarian

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<sup>25</sup>The effect of electoral rules on ideological divergence (polarization) has been thoroughly examined both theoretically (e.g., Cox 1990) and empirically (e.g., Bertelli and Richardson 2008; Calvo and Hellwig 2011). Our approach differs from in the following sense: we are interested in the *reverse direction* of the relationship between ideological divergence and electoral rule disproportionality.

<sup>26</sup>Cox (1990) shows that centripetal incentives are stronger in majoritarian systems while proportional systems are dominated by centrifugal incentives.

systems. Then, adding ideology suggests that a third - and perhaps dominant over the other two (*sufficient electoral uncertainty* and the desire to form a *single-party government*) - condition might affect the prospect of *strategic coordination* among dominant parties in adopting a more majoritarian rule and consolidating the status quo. Hence, exploring how ideology interacts with parties' strategic considerations in the choice of electoral rules might help us understand why electoral rules in many established democracies are relatively stable overtime and why major electoral reforms were (at least until the early 1990s) relatively rare (Katz 2005) and mostly permissive in nature (Colomer 2004). Yet, we defer those questions for future research.

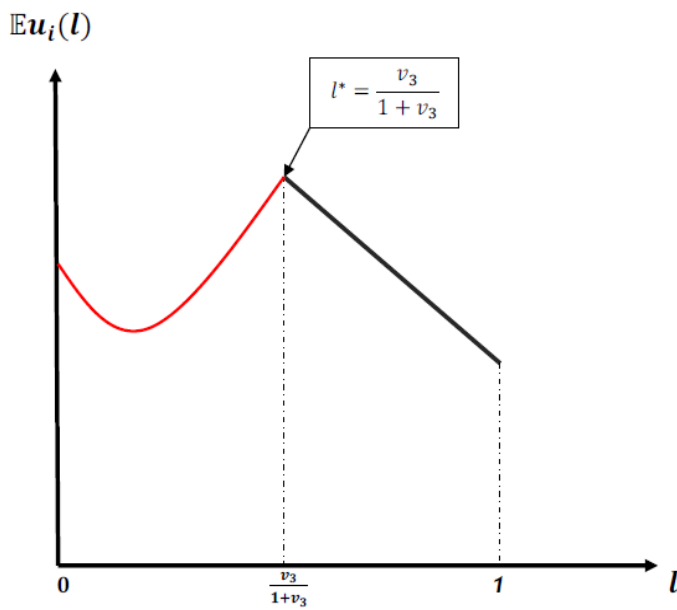
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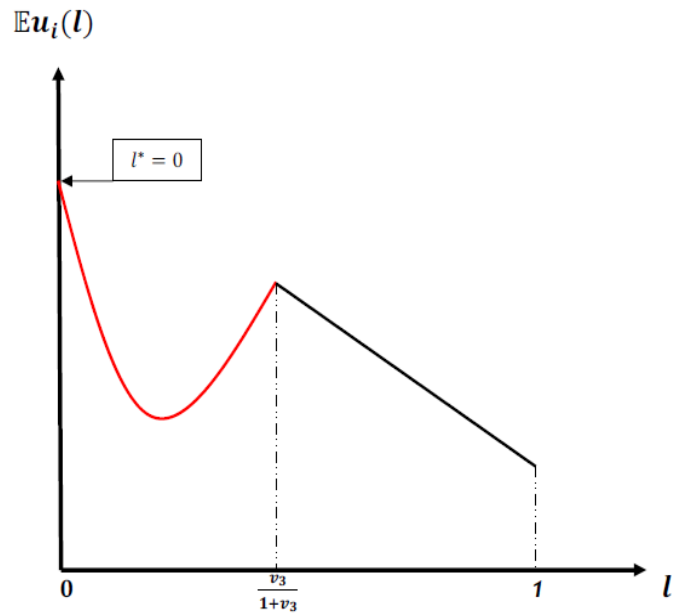
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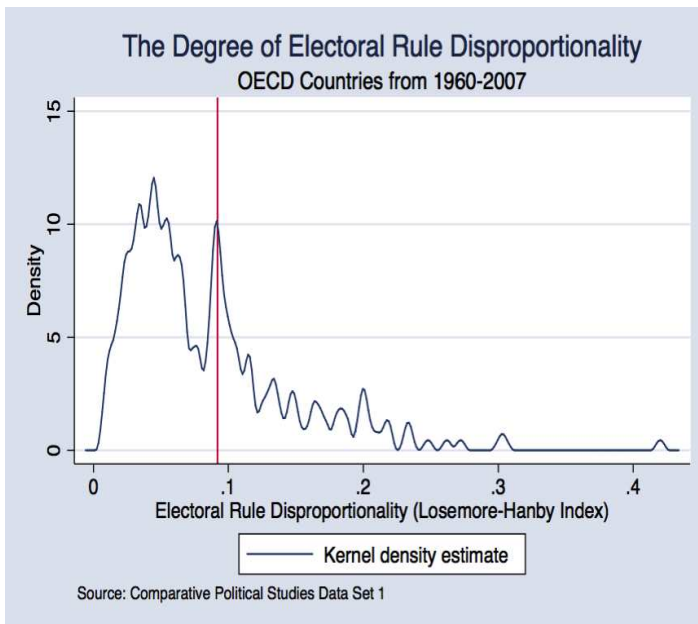
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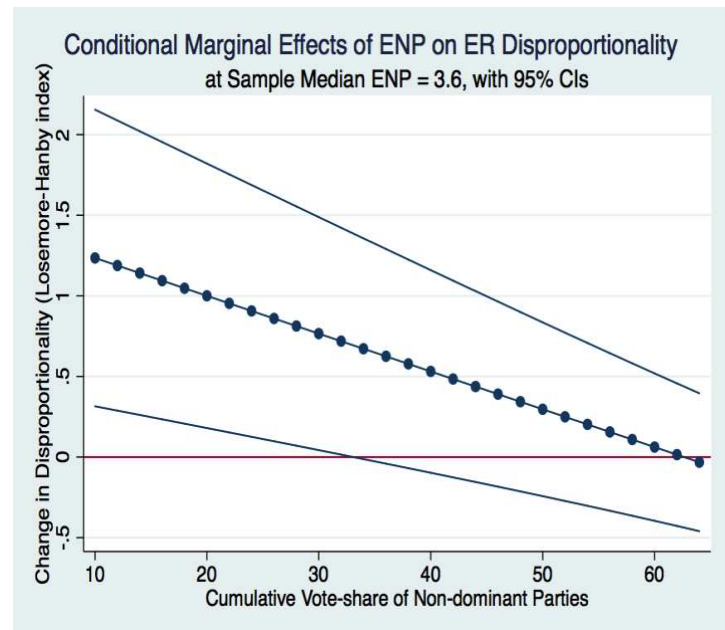
**Figure 1.** Expected Utility as a function of  $l$  for  $i = \{1, 2\}$  when  $l^* = v_3 / (1 + v_3)$ .



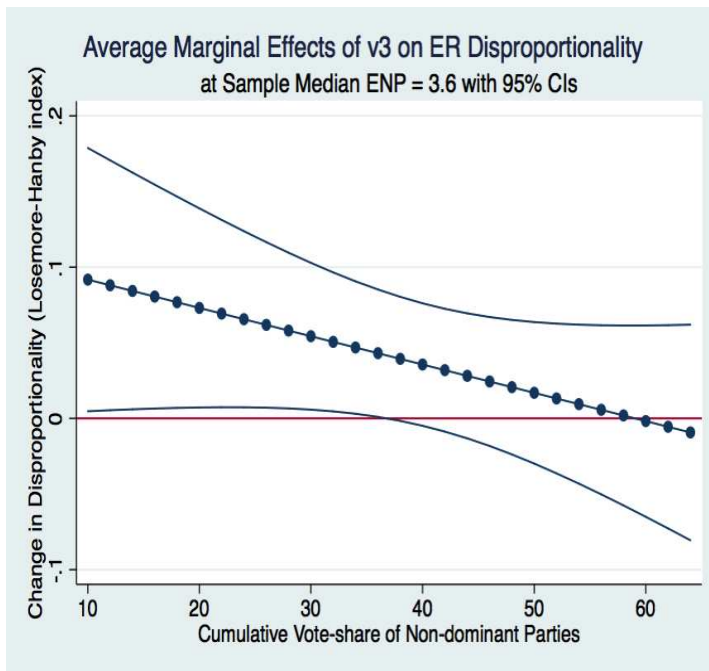
**Figure 2.** Expected Utility as a function of  $l$  for  $i = \{1, 2\}$  when  $l^* = 0$ .



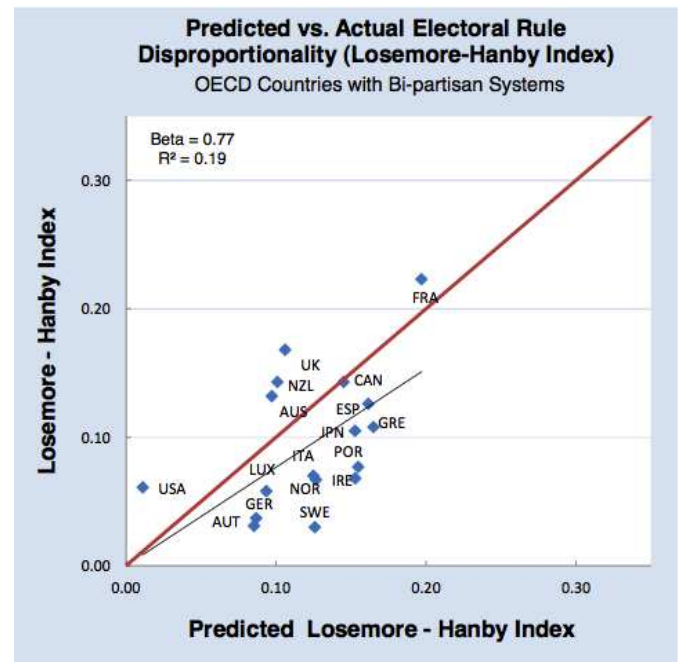
**Figure 3.** Actual electoral rule disproportionality (Losemore-Hanby Index) across OECD countries (1960-2007).



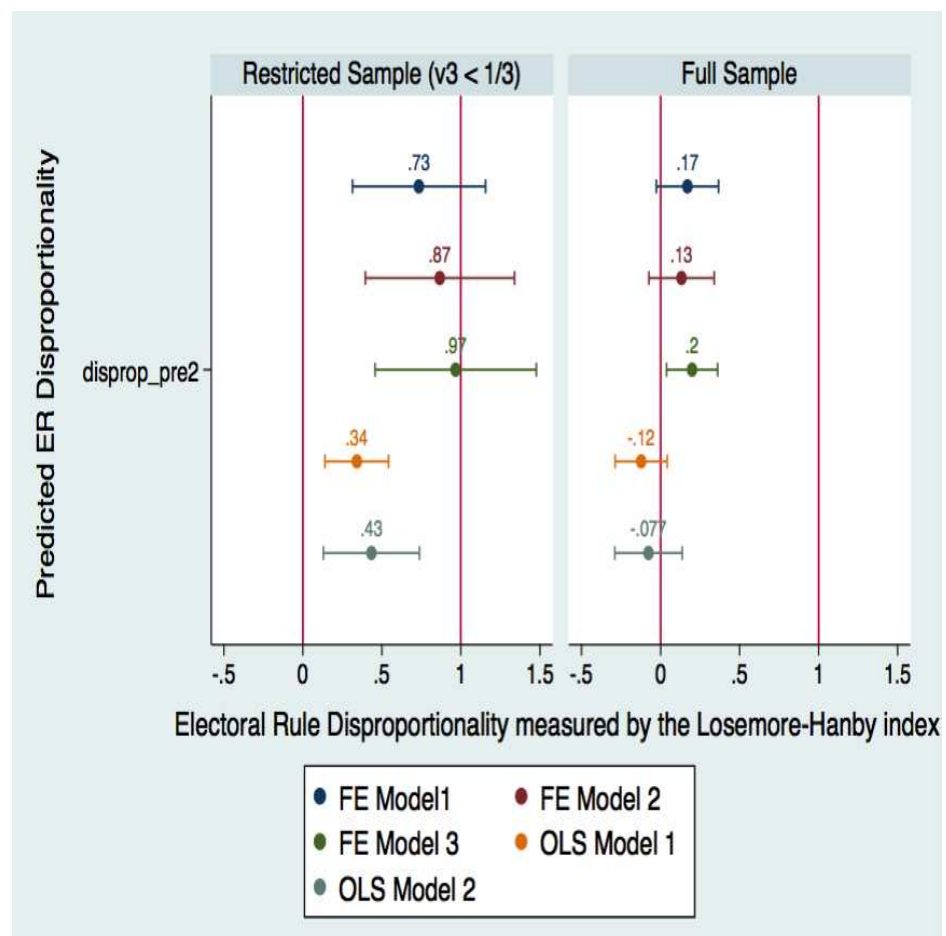
**Figure 4.** Conditional marginal effect of ENP on the disproportionality of the electoral rule (Losemore-Hanby index) for different levels of  $v_3$



**Figure 5.** Average marginal effects of  $v_3$  on the disproportionality of the electoral rule (Losemore-Hanby index).



**Figure 6.** Predicted vs. actual electoral rule disproportionality (Losemore-Hanby index): Aggregate data from 17 OECD countries.



**Figure 7.** Estimating the fit of predicted electoral rule disproportionality (Losemore - Hanby index).



TABLE I. TYPOLOGY OF RECENT MAJOR ELECTORAL REFORMS

Country	Year	Pre-reform rule	Post-reform rule	Type of Reform
Italy (1)	1993	List-PR	MMM	Majoritarian
Italy (2)	2005	MMM	Plurality premium	Majoritarian
France (1)	1958	List-PR	Majority w/ run-off	Majoritarian
France (2)	1987	List-PR	Majority w/ run-off	Majoritarian
Greece (1)	1991	List-PR	Reinforced PR w/ 3% quota	Majoritarian
Greece (2)	2004	Reinforced-PR w/ 3% quota	Plurality premium	Majoritarian
Japan	1994	SNTV	MMM	Majoritarian
Venezuela	1993	List-PR	MMP	Ambiguous
Israel ( <i>a</i> )	1992	List-PR	List-PR w/ quota and majoritarian formula	Majoritarian
Russia	2014	List-PR	MMM	Majoritarian
Romania (1)	1992	List-PR	List-PR w/ 3% quota	Majoritarian
Romania (2)	2000	List-PR w/ 3% quota	List-PR w/ 5% quota (10% for coalitions)	Majoritarian
Romania (3)	2008	List-PR w/ 5% quota (10% for coalitions)	MMM	Majoritarian
Romania (4) ( <i>b</i> )	2012	MMM	FPTP	Majoritarian
Bolivia	1994	List-PR	MMP	Majoritarian
Poland	1993	List-PR (Hare-Niemeyer formula)	List-PR (D'Hondt formula) w/ 5% quota	Majoritarian
New Zealand	1993	FPTP	MMM	Permissive
Georgia	1992	List-PR	MMM	Majoritarian
Hungary	2012	MMP (w/ run-off for SMDs)	MMM (one round for more SMDs)	Majoritarian
Lithuania (1)	1996	MMM (w/ run-off for SMDs and 4% quota)	MMM (w/ run-off for SMDs and 7% quota)	Majoritarian
Lithuania (2)	2000	MMM (w/ run-off for SMDs and 7% quota)	MMM (simple plurality for SMDs & 7% quota)	Majoritarian
Taiwan	2005	SNTV (MMDs) w/ list-PR	MMM	Majoritarian
South Korea	2003	Modified plurality w/ list-PR	MMM	Majoritarian

*Notes:* (*a*) Israel maintained its list-PR with a single nation-wide district but the reform introduced the direct election of the PM which necessitated the use of a majoritarian formula, thus making the system effectively mixed; (*b*) Electoral reform was approved by both Romanian legislative chambers (May 2012) with the support of the two biggest parties but was subsequently rejected by the Constitutional Court.

TABLE II. THE EFFECTS OF A CHANGE IN THE EFFECTIVE NUMBER OF PARTIES INDEX ON THE DISPROPORTIONALITY OF THE ELECTORAL RULE

Dependent Variable	<i>Electoral Rule Disproportionality measured by the Losmore-Hanby index</i>					
	Model 1					
	OLS	OLS	FE	FE	FE	FE
Explanatory Variables	(1)	(2)	(3)	(4)	(5)	(6)
Effective Number Parties ( <i>ENP</i> )	0.001 (0.005)	0.049 (0.018)**	0.012 (0.005)**	0.060 (0.011)***	0.010 (0.004)**	0.070 (0.014)***
Margin of electoral victory (in %)	-0.001 (0.001)	-0.001 (0.001)	0.001 (0.001)	0.002 (0.001)	0.000 (0.001)	0.002 (0.001)
Other controls?	Yes	Yes	No	No	Yes	Yes
Fixed effects?	No	No	Yes	Yes	Yes	Yes
Sample size	Full	$v_3 < 1/3$	Full	$v_3 < 1/3$	Full	$v_3 < 1/3$
$R^2$	0.05	0.26	0.26	0.58	0.29	0.59
$N$	285	151	302	160	285	151

\*  $p < 0.1$ ; \*\*  $p < 0.05$ ; \*\*\*  $p < 0.01$

*Note:* Robust standard errors clustered at the country level reported in parentheses. Unit (country) and time (year) specific fixed effects were included in the specifications of columns 3 to 6. Other controls include: a dummy variable indicating a country with high institutional constraints (see Comparative Political Data Set I for a detailed definition), a dummy variable indicating whether a country has a tradition of coalition governments and a dummy variable indicating the type of democratic regime (parliamentary vs. presidential).

TABLE III. THE CONDITIONAL (ON THE CUMULATIVE VOTE SHARE OF “THIRD” PARTIES -  $v_3$ ) EFFECTS OF A CHANGE IN THE EFFECTIVE NUMBER OF PARTIES ON THE DISPROPORTIONALITY OF THE ELECTORAL RULE

Dependent Variable	<i>Electoral Rule Disproportionality (measured by the Losmore-Hanby index)</i>						
	Model 2					Model 2.b	
	OLS	FE	FE	FE	FE	FE	FE
Explanatory Variables	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Effective Number of Parties ( <i>ENP</i> )	0.035 (0.007)***	0.026 (0.007)***	0.018 (0.008)**	0.028 (0.009)***	0.022 (0.010)**	.-	.-
<i>ENP</i> * $v_3$	-0.041 (0.008)***	-0.014 (0.007)**	-0.007 (0.008)	-0.018 (0.009)**	-0.011 (0.010)	.-	.-
Sum of non-dominant parties vote shares ( $v_3$ )	.-	.-	.-	.-	.-	0.270 (0.076)***	0.275 (0.086)***
$v_3$ - Squared	.-	.-	.-	.-	.-	-0.242 (0.087)***	-0.229 (0.099)**
Additional controls?	No	No	No	Yes	Yes	Yes	Yes
Country fixed effects?	No	Yes	Yes	Yes	Yes	Yes	Yes
Year fixed effects?	No	No	Yes	No	Yes	No	Yes
$R^2$	0.10	0.12	0.26	0.16	0.29	0.15	0.29
$N$	300	300	300	248	248	248	248

\*  $p < 0.1$ ; \*\*  $p < 0.05$ ; \*\*\*  $p < 0.01$

*Note:* Robust standard errors, clustered at the country level, reported in parentheses. A variable indicating the *margin of electoral victory* (in %) and a dummy variable indicating whether a country has *high institutional constraints* (see Comparative Political Data Set I for a detailed definition) are included in all specifications throughout columns 1 to 7. Additional controls (in columns 4 to 7) include: a dummy variable indicating whether a country has a tradition of coalition governments, the number of political parties participating in the cabinet/government, a dummy variable indicating the type of democratic regime (parliamentary vs. presidential system) and a variable measuring the number of years that a country has been an established democracy.

## 6 Appendix A: Proofs

**Lemma 1.** We only need to prove the result for  $i = 1$ . Then, the arguments are equivalent for party 2.

First consider the case of the uniform distribution. If  $b_1 < 1/2$ , then there exist  $\hat{l} \in (0, 1)$  s.t.  $\frac{1/2-\hat{l}}{1-\hat{l}} = b_1$ .

For  $l \in [0, \hat{l}]$  we have that:

$$Eu_1(l) = \frac{1}{(b_1-a_1)} \left[ \int_{a_1}^{(1-v_3)/2} v_1(1-l)dv_1 + \int_{(1-v_3)/2}^{b_1} [v_1(1-l) + l]dv_1 \right].$$

For  $l \in (\hat{l}, \frac{v_3}{1+v_3}]$  we have:

$$Eu_1(l) = \frac{1}{(b_1-a_1)} \left[ \int_{a_1}^{(1-v_3)/2} v_1(1-l)dv_1 + \int_{(1-v_3)/2}^{\frac{1/2-l}{1-l}} [v_1(1-l) + l]dv_1 + \int_{\frac{1/2-l}{1-l}}^{b_1} dv_1 \right].$$

Lastly, for  $l \in (\frac{v_3}{1+v_3}, 1]$  we have:

$$Eu_1(l) = \frac{1}{(b_1-a_1)} \left[ \int_{a_1}^{(1-v_3)/2} v_1(1-l)dv_1 + \int_{(1-v_3)/2}^{b_1} dv_1 \right].$$

One can observe that:  $\frac{\partial^2 Eu_1(l)}{\partial l^2} = 0$ , for  $l \in [0, \hat{l}]$ ;  $\frac{\partial^2 Eu_1(l)}{\partial l^2} > 0$  for  $l \in (\hat{l}, \frac{v_3}{1+v_3}]$ ; and  $\frac{\partial Eu_1(l)}{\partial l} < 0$  for  $l \in (\frac{v_3}{1+v_3}, 1]$ . Moreover, if  $\frac{\partial Eu_1(l)}{\partial l} \geq 0$  for  $l \in [0, \hat{l}]$  then,  $\frac{\partial Eu_1(l)}{\partial l} > 0$  for  $l \in (\hat{l}, \frac{v_3}{1+v_3}]$ . That is,  $Eu_1(l)$  is convex in  $[0, \frac{v_3}{1+v_3}]$ , strictly convex in a subset of  $[0, \frac{v_3}{1+v_3}]$  and decreasing in  $(\frac{v_3}{1+v_3}, 1]$ . If  $b_1 > 1/2$ , for  $l \in [0, \frac{v_3}{1+v_3}]$  we have:

$$Eu_1(l) = \frac{1}{(b_1-a_1)} \left[ \int_{a_1}^{(1-v_3)/2} v_1(1-l)dv_1 + \int_{(1-v_3)/2}^{\frac{1/2-l}{1-l}} [v_1(1-l) + l]dv_1 + \int_{\frac{1/2-l}{1-l}}^{b_1} dv_1 \right]$$

Finally, for  $l \in (\frac{v_3}{1+v_3}, 1]$  we have:

$$Eu_1(l) = \frac{1}{(b_1-a_1)} \left[ \int_{a_1}^{(1-v_3)/2} v_1(1-l)dv_1 + \int_{(1-v_3)/2}^{b_1} dv_1 \right].$$

Just as before  $\frac{\partial^2 Eu_1(l)}{\partial l^2} > 0$  for  $l \in [0, \frac{v_3}{1+v_3}]$ , and  $\frac{\partial Eu_1(l)}{\partial l} < 0$  for  $l \in (\frac{v_3}{1+v_3}, 1]$ . That is,  $Eu_1(l)$  is strictly convex in  $[0, \frac{v_3}{1+v_3}]$  and decreasing in  $(\frac{v_3}{1+v_3}, 1]$ .

Now consider the case of a more general class of distributions  $\Phi(\cdot)$  and let  $b_1 < 1/2$ . Then, for  $l \in [0, \hat{l}]$  we have that:

$$Eu_1(l) = \int_{a_1}^{(1-v_3)/2} v_1(1-l)d\Phi(v_1) + \int_{(1-v_3)/2}^{b_1} [v_1(1-l) + l]d\Phi(v_1).$$

For  $l \in (\hat{l}, \frac{v_3}{1+v_3}]$  we have:

$$Eu_1(l) = \int_{a_1}^{(1-v_3)/2} v_1(1-l)d\Phi(v_1) + \int_{(1-v_3)/2}^{\frac{1/2-l}{1-l}} [v_1(1-l) + l]d\Phi(v_1) + \int_{\frac{1/2-l}{1-l}}^{b_1} d\Phi(v_1).$$

Finally, for  $l \in (\frac{v_3}{1+v_3}, 1]$  we have:

$$Eu_1(l) = \int_{a_1}^{(1-v_3)/2} v_1(1-l)d\Phi(v_1) + \int_{(1-v_3)/2}^{b_1} d\Phi(v_1).$$

Observe again that for  $l \in [0, \hat{l}]$ , we have  $\frac{\partial^2 Eu_1(l)}{\partial l^2} = 0$ ; for  $l \in (\hat{l}, \frac{v_3}{1+v_3}]$ , we have  $\frac{\partial^2 Eu_1(l)}{\partial l^2} > 0$  iff  $2(l-1)\Phi'(\frac{2l-1}{2l-2}) + \Phi''(\frac{2l-1}{2l-2}) < 0$ , while for  $l \in (\frac{v_3}{1+v_3}, 1]$  we have  $\frac{\partial Eu_1(l)}{\partial l} < 0$ . Given that  $\frac{\partial Eu_1(l)}{\partial l} < 0$  for

$l \in (\frac{v_3}{1+v_3}, 1]$  the equilibrium electoral rule bonus for any  $\Phi(\cdot)$  is always smaller or equal to  $\frac{v_3}{1+v_3}$ . Moreover, it is easily checked that a sufficient (but not necessary) condition for the inequality  $2(l-1)\Phi'(\frac{2l-1}{2l-2}) + \Phi''(\frac{2l-1}{2l-2}) < 0$  to be satisfied is for the density of  $\Phi(\cdot)$  to be weakly concave (or at least not too convex). It is, then, easily checked that this sufficient condition is satisfied by any distribution with a linear density (independently of whether it is increasing or decreasing), the truncated Normal, the truncated Beta, the semicircle and other distributions. Hence, we have shown that for a large class of  $\Phi(\cdot)$ 's expected utility is convex in  $[0, \frac{v_3}{1+v_3}]$ . Then in those cases as well, the results are guaranteed to be identical to the case of uniform  $\Phi(\cdot)$  that we analyze in the paper. That is, that the only values of  $l$  that are candidates for maximum are either 0, or  $\frac{v_3}{1+v_3}$ . This concludes the proof. ■

**Proposition 1.** When the electoral rule reform is trivial then this implies that the proposer faces an unconstrained maximization problem. Hence for a change of the electoral rule from 0 to  $\frac{v_3}{1+v_3}$  it suffices to show that  $Eu_1(l = \frac{v_3}{1+v_3}) > Eu_1(l = 0)$ , that is  $f_1(a_1, b_1, v_3) > 0$ , which in turn implies that  $(b_1 - \frac{1-v_3}{2})(2 - b_1 - \frac{1-v_3}{2}) > (\frac{v_3}{1+v_3})(\frac{1-v_3}{2} + a_1)(\frac{1-v_3}{2} - a_1)$ . In the reverse direction, assume that the above inequality holds true but no electoral rule change takes place, that is  $l = 0$ . But then it must be that  $Eu_1(l = \frac{v_3}{1+v_3}) < Eu_1(l = 0)$ , a clear contradiction. The *only if* part follows. ■

**Lemma 2.** Since the third party expects to receive the premium  $l$  with probability zero, it strictly prefers the PR rule ( $l = 0$ ). Its expected utility from any electoral rule  $l \neq 0$  is  $Eu_3(v_3, l) = v_3(1 - l)$ , which is strictly decreasing in  $l$ . ■

**Lemma 3.** Note that, in the symmetric case we always have  $\frac{a_1+b_1}{2} = \frac{a_2+b_2}{2} = \frac{1-v_3}{2}$ . In such a case it can be easily checked algebraically that  $f_i(\cdot) = \frac{1+2a_i+v_3(3+4v_3)}{4(1+v_3)} > 0$  for every  $b_i < \frac{1}{2}$ , for  $i \in \{1, 2\}$  (for  $b_i \geq \frac{1}{2}$  we know that  $f_i(\cdot) > 0$  independently of whether we are in the symmetric case or not). ■

**Proposition 2.** Notice that we can always find  $\delta$  such that we can rewrite  $a_i = a_{-i} + \delta$ . Then,  $b_i = 1 - a_{-i} - v_3 = 1 - a_i - v_3 + \delta$ , for  $i \in \{1, 2\}$ . Again we assume that  $b_i < \frac{1}{2}$  because otherwise  $f_i(\cdot) > 0$  trivially holds. Now, we can express condition  $f_i(a_i, b_i, v_3) > 0$  in terms of  $a_i, v_3$  and  $\delta$ , where  $\delta$  measures the degree of asymmetry. Then, we can rewrite the condition  $f_i(\cdot) = (b_i - \frac{1-v_3}{2})(2 - b_i - \frac{1-v_3}{2}) - (\frac{v_3}{1+v_3})(\frac{1-v_3}{2} + a_i)(\frac{1-v_3}{2} - a_i) > 0$  as:

$$\left( \frac{1}{1-2a_i-v_3-\delta} \right) \left[ \frac{1-4a_i^2+v_3(2+v_3-4v_3^2)-8v_3(1+v_3)\delta-4(1+v_3)\delta^2-8a_i(1+v_3)(v_3+\delta)}{4(1+v_3)} \right] > 0$$

But, observe that in the symmetric case ( $\delta = 0$ ) the above condition becomes  $\frac{1+2a_i+v_3(3+4v_3)}{4(1+v_3)} > 0$  which is always satisfied, since the *LHS* is *strictly positive* for any  $a_i, v_3$  (Lemma 3). Hence, by continuity of  $f_i(\cdot)$  in  $\delta$ , we can always find  $z > 0$  such that for every  $\delta \in (-z, z)$ ,  $f_i(\cdot)$  is *strictly positive*, even if  $i$  is *the expected loser* (that is  $\delta < 0$ ), as was the case in our example. This completes the proof. ■

## 7 Appendix B

### 7.1 General Model

Assume that:

$$u_i(v_1, v_2, v_3, l) = \begin{cases} s_i^l(v_i) & , \text{ if } s_i^l(v_i) \leq 1/2 - s_3^l(v_3) \\ s_i^l(v_i) + r_1 & , \text{ if } 1/2 - s_3^l(v_3) \leq s_i^l(v_i) \leq 1/2 \\ gs_i^l(v_i) + r_2 & , \text{ if } s_i^l(v_i) > 1/2 \end{cases}$$

where  $0 \leq r_1 < \frac{1}{2} + r_1 < r_2$  and  $g \in [0, 1]$  and that  $a_1 = a_2 < \frac{1}{2} < b_1 = b_2$  (symmetric parties). That is, the utility function analyzed extensively in the paper is directly derived from the above formulation if one sets  $r_1 = g = 0$  and  $r_2 = 1$ .

The first part is the utility of party  $i$  when it stays in opposition and some other party forms a single-party government. In this case,  $i$ 's utility depends only on the number of its seats. The second part is the utility of party  $i$  when no party can form a single-party government and as a result, a coalition government is formed. Thus, on top of its seat share party  $i$  receives extra rents  $r_1$  from holding office. Finally, the third part gives the utility of party  $i$  when it forms a single-party government. In this case, the payoff of the party depends, as before, on its seat share and on office rents  $r_2$  (which are obviously higher than the rents  $i$  gets in a coalition government). Parties in this environment care about forming a single-party government ( $\frac{1}{2} + r_1 < r_2$ ). Moreover,  $g \in [0, 1]$  implies that the utility from an extra seat when the party is not able to form a single-party government ( $s_i^l(v_i) \leq 1/2$ ), is generically higher than the utility from an extra seat when the party is able to form a single-party government.<sup>27</sup> Figure 10 (Appendix B) depicts such a utility function.

Further notice that  $s_i^l(v_i) = v_i(1-l)$  when  $v_i < (1-v_3)/2$ ,  $s_i^l(v_i) = v_i(1-l) + \frac{l}{2}$  when  $v_i = (1-v_3)/2$  (parties 1 and 2 split the bonus when they tie) and that  $s_i^l(v_i) = v_i(1-l) + l$  when  $v_i > (1-v_3)/2$ . Therefore, the expected utility of party 1 (since parties 1 and 2 are symmetric we derive the outcome of the game only by studying party 1) from an electoral rule  $l \leq \frac{v_3}{1+v_3}$  is given by:

$$\begin{aligned} Eu_1(l) = & \frac{1}{(b_1-a_1)} \left[ \int_{a_1}^{\frac{1}{2(1-l)}-v_3} v_1(1-l) dv_1 + \int_{\frac{1}{2(1-l)}-v_3}^{(1-v_3)/2} [v_1(1-l) + r_1] dv_1 + \right. \\ & \left. + \int_{(1-v_3)/2}^{\frac{1/2-l}{1-l}} [v_1(1-l) + r_1 + l] dv_1 + \int_{\frac{1/2-l}{1-l}}^{b_1} [g(v_1(1-l) + l) + r_2] dv_1 \right] \end{aligned}$$

<sup>27</sup> A party derives utility from an extra seat in two ways: directly, and indirectly (by increasing its chances of forming a single-party government). Once its seat share exceeds the parliamentary majority threshold, the second channel disappears. Hence, the marginal utility of an extra seat might decrease.

and from an electoral rule  $l > \frac{v_3}{1+v_3}$  is given by:

$$Eu_1(l) = \frac{1}{(b_1-a_1)} \left[ \int_{a_1}^{(1-v_3)/2} v_1(1-l)dv_1 + \int_{(1-v_3)/2}^{b_1} [g(v_1(1-l) + l) + r_2]dv_1 \right].$$

We compute:

$$Eu_1(l=0) = \frac{-1+4a_1^2+g-4b_1^2g-8r_1v_3+4r_2-8b_1r_2}{8(a_1-b_1)}$$

and

$$Eu_1(l = \frac{v_3}{1+v_3}) = \frac{-1+4a_1^2+g-4b_1^2g+4l-4a_1^2l+4gl-8b_1gl+4b_1^2gl-v_3-3gv_3+4r_2-8b_1r_2-4v_3r_2}{8(a_1-b_1)}.$$

Without loss of generality we normalize  $r_1 = 1$  and we find that  $Eu_1(l = \frac{v_3}{1+v_3}) > Eu_1(l=0)$  holds if and only if:

$$r_2 > \frac{7+8b_1-4b_1^2+g-8b_1g+4b_1^2g+15v_3-8b_1v_3-3gv_3-4v_3^2}{4+4v_3}$$

and that  $\frac{\partial^2 Eu_1(l)}{\partial l^2} > 0$  for  $l \leq \frac{v_3}{1+v_3}$  holds if and only if:

$$r_2 > \frac{1}{4}(11-3g).$$

Moreover we observe that  $\frac{\partial Eu_1(l)}{\partial l} < 0$  for  $l > \frac{v_3}{1+v_3}$  occurs if and only if:

$$g < \frac{-3+2b_1+3v_3}{-3+2b_1-v_3}.$$

That is, for sufficiently high  $r_2$  (sufficiently high rents from forming a single-party government) and for sufficiently low  $g$  (sufficiently low gain from an extra seat when the party already has a majority of votes in the parliament) both parties maximize their expected utilities by applying the  $l = \frac{v_3}{1+v_3}$  electoral rule.

To conclude let us state that there are three more possibilities. For  $r_2$  sufficiently low the most probable outcome is that parties will choose  $l = 0$  (PR). There is a very small set of parameter values,<sup>28</sup> given by:

$$\frac{7+8b_1-4b_1^2+g-8b_1g+4b_1^2g+4v_3-8b_1v_3-4v_3^2}{4} < r_2 < \frac{7+8b_1-4b_1^2+g-8b_1g+4b_1^2g+26v_3-8b_1v_3-6gv_3+7v_3^2-3gv_3^2}{4+8v_3+4v_3^2}$$

that leads parties to adopt an electoral rule  $l \in (0, \frac{v_3}{1+v_3})$  such that:

$$l = 1 - \frac{\sqrt{11a_1^2-11g-3a_1^2g+22b_1g-11b_1^2g+3g^2-6b_1g^2+3b_1^2g^2+11v_3-3gv_3-4a_1^2r_2+4gr_2-8b_1gr_2+4b_1^2gr_2-4v_3r_2}}{2(a_1^2-g+2b_1g-b_1^2g+v_3)}.$$

Finally, in the extreme case where both  $r_2$  and  $g$  are very large, the first-past-the-post electoral rule ( $l = 1$ ) may be selected by the parties.

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<sup>28</sup>This claim is derived from parametric analysis using *Mathematica*.

## 7.2 Insufficient Electoral Uncertainty

When the electoral competition is trivial we can state the following results.

**Proposition 3** *When both the electoral reform process and the electoral competition are trivial and the proposer is: (i) the leading party, then  $l^* \geq \max\{0, \frac{1/2-a_1}{1-a_1}\}$ , (ii) not the leading party, then  $l^* = 0$ .*

**Proof.** Since the electoral reform process is trivial, Party 1 faces an unconstrained maximization program. Moreover, since the electoral competition is trivial, if it is the leading party, its expected utility is given by:  $Eu_1(l) = \frac{1}{(b_1-a_1)} \left[ \int_{a_1}^{\frac{1/2-l}{1-l}} [v_1(1-l) + l] dv_1 + \int_{\frac{1/2-l}{1-l}}^{b_1} dv_1 \right]$ . Then, it proposes  $l^*$  such that it secures with certainty the majority of the seats in the new parliament. That is, it sets  $a_1(1-l^*) + l^* = 1/2$ , which implies that  $s_1^{l^*} = 1/2$ ,  $\forall a_1$ . Then, solving for  $l^*$  yields the result. If it is not the leading party,  $Eu_1(l) = \frac{1}{(b_1-a_1)} \left[ \int_{a_1}^{b_1} v_1(1-l) dv_1 \right]$ , which is strictly decreasing in  $l$ . Hence,  $l^* = 0$ . This completes the proof. ■

This result can be viewed as the simplest case scenario. The idea behind this proposition is very simple. Since the electoral reform process is trivial, the proposer holds enough seats in the current parliament to enact any electoral rule reform, without the consent of Party 2. Hence, the proposer just chooses  $l^*$  in order to maximize expected utility. Given that the electoral competition is trivial, if the Proposer is not the leading party (i.e. sure loser) it proposes that the PR rule is not amended ( $l^* = 0$ ). As a result, the electoral rule does not change. Otherwise, it proposes  $l^* \geq \max\{0, \frac{1/2-a_1}{1-a_1}\}$ . That is, if Party 1 is the leading party (i.e. sure winner), the proposed proportionality distortion  $l^*$  will be such that it will guarantee the formation of a single-party government by Party 1. The solution to this unconstrained maximization problem yields with certainty the majority of the seats in the parliament for every possible realization of  $a_1$ . Hence, by choosing this level of  $l^*$  the leading party ensures the highest possible level of utility. On the other hand, if it is not the leading party it can never get the premium  $l^*$ , making its utility strictly decreasing on  $l^*$ . Hence, any distortion to the PR rule is not desirable.

**Proposition 4** *When the electoral competition is trivial but the electoral reform process is non-trivial, then  $l^* = 0$ .*

**Proof.** Since electoral competition is trivial one party is a sure loser and the other is a sure winner. That is, for  $i = 1$  or  $2$ ,  $Eu_i(l) = \frac{1}{(b_1-a_1)} \left[ \int_{a_1}^{b_1} v_i(1-l) dv_i \right]$ , which is strictly decreasing in  $l$ . Hence, one party always prefers  $l^* = 0$  but the other, as shown in Proposition 4 prefers  $l^* \geq \max\{0, \frac{1/2-a_1}{1-a_1}\}$ . Since the



electoral reform process is non-trivial, the two parties have to agree on the electoral reform. So, the only equilibrium is  $l^* = 0$ . ■

The idea that drives the result is that in this case there is no room for collusion. The strategic incentives of the two dominant parties do not align, because there is no uncertainty over the outcome of the electoral competition. The leading party will always prefer a value of  $l^* > 0$  but the other party will always reject this proposal because its utility is strictly decreasing in  $l$ . Since the electoral reform is non-trivial and requires the consent of both dominant parties, it is obvious why no electoral rule reform will ever be enacted by this parliament. That is, the status-quo is maintained and PR persists as the electoral rule (i.e.  $l^* = 0$ ).

### 7.3 Additional Empirical Results: Goodness of Fit

In the paper we have presented some aggregate level evidence in support of the goodness of fit between our theoretical predictions and the actual data. Despite this being only a rough approximation, since it is true that in some countries the two dominant parties have not been historically perfectly symmetric in terms of their vote shares, the fit of the predicted *Losemore-Hanby* index against the actual disproportionality of the electoral rule appeared to be quite good. Here, we estimate the complete model on the full sample that contains individual observations (at the country-year level). That is, we estimate the following equation (Model 3):

$$LH_{n,t} = \beta_0 + \beta_1 \widehat{LH}_{n,t} + \mathbf{X}'_{n,t} * \boldsymbol{\gamma} + \alpha_n + \lambda_t + \epsilon_{n,t} \quad (3)$$

where  $\widehat{LH}_{n,t}$  is the predicted value of the  $LH$  index of disproportionality (and under the assumptions specified above was calculated to be equal to  $\frac{(v_3)_{n,t}}{2}$ ) for country  $n$  in election  $t$  and all the other variables are as defined above. Here, it may worth noting that the above specification is directly analogous the specification of equation (1) once we replace  $ENP$  with  $v_3$ . This is so, because the predicted  $\widehat{LH}_{n,t}$  index (which we use as our independent variable in the equation above) is clearly a strictly and monotonically increasing function of  $v_3$  and, moreover, it *does not* vary in the way that the  $v_3$  is distributed among smaller parties.<sup>29</sup> As a result, Model 3 is a direct substitute for Model 1.

By observing equation (3) it becomes apparent that a perfect fit of our calibrated  $\widehat{LH}_{n,t}$  to the actual data would imply a coefficient value for  $\beta_1 = 1$ , controlling for everything else. We present these results

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<sup>29</sup> Recall that, in the symmetric case, our model predicts that  $l = \frac{v_3}{1+v_3}$  and by replacing this value to the Losemore-Hanby formula we can get its predicted value  $\widehat{LH}(\frac{v_3}{1+v_3}) = \frac{v_3}{2}$ . Then, we can substitute  $v_3$  with the sum of vote shares of the remaining non-dominant parties in country  $n$  in election  $t$ , in order to obtain a predicted value for  $\widehat{LH}_{n,t}$ .

in Table 4 (Appendix B). In all specifications (especially so, when  $v_3 < 1/3$ ) there is a positive and statistically significant (at any conventional level) correlation between the predicted and the actual values of the  $LH$  index (the coefficient ranges from .043 to almost 1). Moreover in columns 3, 5 and 7 when we restrict our attention to the sub-sample where  $v_3 < 1/3$  (recall that we have implicitly assumed  $v_3 < 1/3$  when we calculated  $\widehat{LH}_{n,t}$  for  $l = \frac{v_3}{1+v_3}$ ) the estimate of the fit is *statistically indistinguishable* from one (ranging from 0.74 to 0.97) at any conventional level of significance. In fact, if one looks at the visual summary of our results (Fig. 7) it becomes clear that the predicted  $LH$  index is statistically indistinguishable from the actual index (that is, the value of  $\beta_1$  is not statistically different from one) in almost all specifications.<sup>30</sup> Hence, we conclude that the goodness of the fit is satisfactory not only at the aggregate but also at the individual (country-year) level. In sum, the comparative statics exercise that we have conducted above has shown that (in equilibrium) the perturbations on the degree and the direction of the disproportionality of the electoral rule, measured by the *Losemore-Hanby* index, are consistent with the predictions of our formal model.

## 7.4 Case-study Analysis

In order to identify whether empirical patterns of electoral rule change fit our model's description, we will examine a series of case-studies.

### 7.4.1 Greece

We will first examine the case of recent electoral reforms in Greece. Greece underwent four major electoral rule reforms in the period under consideration, namely in 1988, 1991, 2004 and 2006. Table 5 (Appendix B) presents the actual electoral results, the vote and seat share allocations, the actual plurality premium,  $\tilde{l}$ ,<sup>31</sup> and the predicted one ( $l = \frac{v_3}{1+v_3}$ ). We also report the observed deviations between the actual and the predicted values. In fact, our model seems to perform quite well in predicting the direction of electoral rule change in Greece. As one can observe in Figures 8 and 9 (Appendix B), from 1981-1985 the actual electoral rule is more disproportional than the optimum. Yet, the first electoral reform (1988) introduces the PR rule which results in more proportionality than it desired by the dominant parties. As a result, another reform follows (1991) that makes again the rule more majoritarian. During the third electoral

<sup>30</sup>Moreover, note that when  $v_3 < 1/3$  the predicted values of the  $LH$  index are always positive and statistically different from zero (at any conventional level).

<sup>31</sup>Our model can accommodate a continuum of electoral rules based on their disproportionality. One need only compute  $\tilde{l} = \frac{\tilde{s}_1 - \tilde{v}_1}{1 - \tilde{v}_1}$  in order to find the actual premium  $\tilde{l}$ , which corresponds to the value of the premium when we allow the actual seat allocation to be replicated by our model.

reform (2004), which was first applied in the 2007 elections, a 40-seat plurality premium is introduced, quite close to the predicted optimum. Finally, Greek Parliament amended the rule in 2006 introducing a 50-seat premium, in accordance with the theoretical predictions of the model. Moreover, with the exception of the 1989 elections where a more permissive rule was applied, the observed gap between the actual and optimal plurality premium is shrinking over time. Overall, we observe that the actual and predicted values are moving in the same direction and do not differ significantly from each other. That is, both dominant parties in Greece have been quite strategic in choosing the “right” electoral rule, in order to consolidate the bipartisan system and avoid political power-sharing with smaller parties (single-party governments).

Furthermore, the permissive reform of 1988 provides further justification on our decision to focus on restrictive reforms which are non-trivial in terms of strategic coordination. Back in 1998 it was widely accepted that the conservative opposition will win the forthcoming 1998 elections by a significant margin over the socialist government. As a result the socialists which enjoyed a comfortable majority in the 1998 parliament single-handedly passed a permissive electoral reform (introducing pure list-PR) in order to mitigate their expected loss in seats. Nevertheless, this electoral reform, which is also consistent with our model’s predictions is a trivial one, both in terms of the process and also with respect to the electoral competition. As a result, it boils down to a simple utility (seat) maximization problem that a single party faces (see Proposition 1).

#### 7.4.2 Poland

But in addition to Greece, which admittedly has one of the most fluctuating electoral system among the advanced OECD countries, we also examine additional cases of electoral rule reform in newly-established democracies in east Europe (e.g., Poland, Lithuania) where many of the most recent reforms have taken place. For instance, the restrictive electoral reform in Poland that took place in 1993 is a good example to illustrate our theory.<sup>32</sup> As Benoit and Hayden (2004) document the decision of the Polish parliament to replace the list-PR rule (applying the Hare formula which is deemed to be the most proportional) with a list-PR rule that applied the d’Hondt formula (less proportional) and introduce a 5% exclusion quota (reaching up to 8% for coalitions) was mainly driven by the strategic coordination and calculations of the two biggest parties (the Democratic Union/UD, a pro-Solidarity liberal party, and the Alliance of

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<sup>32</sup>Benoit and Hayden (2004) provide a very detailed account of all the episodes of electoral reform in Poland in the early 1990s. They analytically show, by compiling data on actual and expected (by collecting a series of public opinion surveys before the election) electoral and parliamentary strength for each party, that electoral reform in Poland was driven by opportunistic seat-maximizing parties.

Democratic Left/SLD, a second-generation post-communist party) which were polling high and were expected to do well in the forthcoming elections, thus maintaining their dominant position.

Moreover, the near-change of the electoral rule in 1997 offers additional support to our model. This failed attempt to introduce a more permissive electoral rule was blocked by the strategic coordination between the two dominant parties, the Alliance of Democratic Left/SLD and the Solidarity Electoral Action, which coalesced despite their obvious political differences in order to maintain the current status-quo that suited their interests (the Alliance was the largest party in the current Parliament and its support was steadily rising and so did support for the Solidarity which was also expected to do equally well in polls and was contesting the election -in fact it won). Hence, as both dominant parties opposed the more permissive reform, and given that they had enough seats in the Parliament (a result of the less proportional rule of 1993) to block it, no reform took place.<sup>33</sup> The same scenario (but with a different outcome) was replayed in the successful electoral reform of 2001, where the 1993 rule was substituted by a more permissive one (applying the Sainte-Laguë formula with no exclusion quota). But this time, the winner of previous elections the Solidarity Electoral Action (that had received almost a third of the votes and was the largest party in the Parliament, followed by the SLD the other dominant party) was expected to perform very poorly in the forthcoming elections and lose more than half of its electoral power. And, since it had the largest group in the Parliament, this time it coalesced with the smaller parties (which preferred a more permissive rule) in order to pass the reform.<sup>34</sup> Hence, this permissive reform strengthens our argument on the strategic triviality of permissive reforms as opposed to majoritarian ones. The former, as the 1997/2001 incidence revealed, occurred once the largest parliamentary party (the Solidarity Electoral Action in this case) was expected to suffer big losses in the forthcoming elections. The latter, instead, required strategic cooperation among the two dominant parties (and sufficient electoral uncertainty) in order to be enacted, exactly as our model suggests. This is perhaps one of the reasons why majoritarian reforms are less likely to occur (as strategic alignment is not always guaranteed) but also constitute the most interesting cases as well.

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<sup>33</sup>As Benoit and Hayden (2004) note “[...] the ‘Against’ side [...] paired essentially the two key rival forces in the party system.” This odd alliance was formed for no other reason than pure opportunistic motivations: to maintain the status-quo that served them both.

<sup>34</sup>As Benoit and Hayden (2004) note “[ASW’s] actual seats made [it] the largest group in the Sejm [Parliament], but according to the polls [...] [they] know that a non-proportional electoral law will destroy them.”

### 7.4.3 Lithuania and Romania

Nevertheless, Poland is not the only case among the newly-established east European democracies that offers a test-bed for our theory. A similar pattern emerges if one studies the cases of electoral reforms in Lithuania and Romania. For instance in Romania, a series of majoritarian electoral reforms that took place in the last two decades are also consistent with our theoretical framework. After a very brief post-communist transition period where a pure list-PR rule was applied, as Pilet and De Waele (2007) note, “[...] the story of the Romanian electoral system is the story of a movement to make the electoral law less inclusive.” This attempt included the introduction of exclusion quotas (1992) which have been subsequently raised (2000) and a series of (successful) proposals to change the rule into a more majoritarian one (2008) which eventually gained ground among Romanian politicians representing the two dominant parties (the post-communist Social Democratic Party and the National Liberal Alliance). Moreover, in all those instances the authors conclude that “[...] the main motivation pushing for a less inclusive electoral law [...] appears [to be] that the largest Romanian parties hope the reform will strengthen their political representation.” Pilet and De Waele (2007) also conclude that increased electoral uncertainty and the strategic incentives (and coordination) of the two big parties were the drivers behind electoral rule choice in Romania. Four years later (2012), the circumstances under which the Mixed-Member (MM) electoral system was replaced by the even more majoritarian (and more restrictive) UK-style, first-past-the-post (FPTP) system with single-member districts are strikingly similar. Supported by both dominant parties, which were both expected to well in the forthcoming elections and maintain their positions, the new electoral rule was approved by both chambers of the Romanian Parliament.<sup>35</sup>

A similar pattern also emerges if one is to consider the two majoritarian electoral reforms that took place in Lithuania in 1996 and 2000, respectively. As Martinaitis (2012) notes, in both instances the electoral rule became more restrictive as a result of the strategic coordination of the two dominant political blocs which “[...] sought to avoid competition from smaller ‘challenger’ parties by reducing the proportionality of the mixed electoral system between 1992 and 2000.” Hence, both the pattern of electoral reform in Lithuania throughout the 1990s, as well as the series of electoral reforms in Romania over the last two decades appear to be consistent with the theoretical framework that we have developed.

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<sup>35</sup>The reform was approved both by Romania’s Senate and the Chamber of Deputies in May 2012, with the support of the two bigger parties. Despite being later rejected by the Constitutional Court, the acceptance of the new rule by both chambers in the parliament shows the strategic coordination between the two dominant parties.

#### 7.4.4 Taiwan and South Korea

But it is not only in east Europe that those patterns of electoral reform have emerged. The 2005 electoral reform in Taiwan, where the semi-proportional SNTV system (with multi-member districts and a list-PR allocation) was replaced by the more majoritarian, parallel Mixed-Member system (with single-member districts decided by simple plurality and no compensatory link between the nominal and the list tiers), in another example of strategic collusion between the two dominant parties. As Stockton (2009) notes, in the case of Taiwan, “[...] reform was the result of collaboration between arch political rivals, the KMT [Chinese Nationalist Party] and DPP [Democratic Progressive Party]” in an attempt to diminish the influence of smaller parties and the incidence of coalition governments. In fact, both parties desired a more majoritarian system and agreed to introduce a “large party bonus” as they both sought to create a two-party system. A similar pattern appears in the majoritarian reform in South Korea (2003) which also introduced the “parallel” Mixed-Member system. In this instance, as well, the reform was driven once more by the dominant parties and their desire to create a majoritarian two-party system and consolidate their position. Overall, in both cases the role of dominant parties, without whom those majoritarian reforms would not have been possible, was crucial in understanding the dynamics behind this wave of majoritarian reforms (Stockton 2009). As a result, this move to more majoritarian systems has given rise to single party majorities in Taiwan and South Korea, as the old dominant parties had regained majority status in their respective national assemblies by 2008 (Stockton 2009).

#### 7.4.5 Italy

Finally, two attempts for introducing a more majoritarian electoral rule in Italy, in 1999 and 2014 respectively, are also characterized by the same strategic motivations of dominant parties as outlined above. In the first instance (1999) it was a failed attempt to replace the Mixed-Member proportional electoral rule (with a compensatory PR-list) with a pure FPTP system with single-member districts. This attempt was strongly supported by both dominant Italian political parties as it was deemed that it would benefit them greatly but was defeated in a referendum.<sup>36</sup> In any case, despite its narrow defeat, this attempted reform to substitute the MMP system with the FPTP was suggestive of the incentives of the dominant parties to coordinate in order to consolidate their positions and reduce the impact that smaller parties have in government formation. A similar logic is behind the attempted current electoral reform. Ital-

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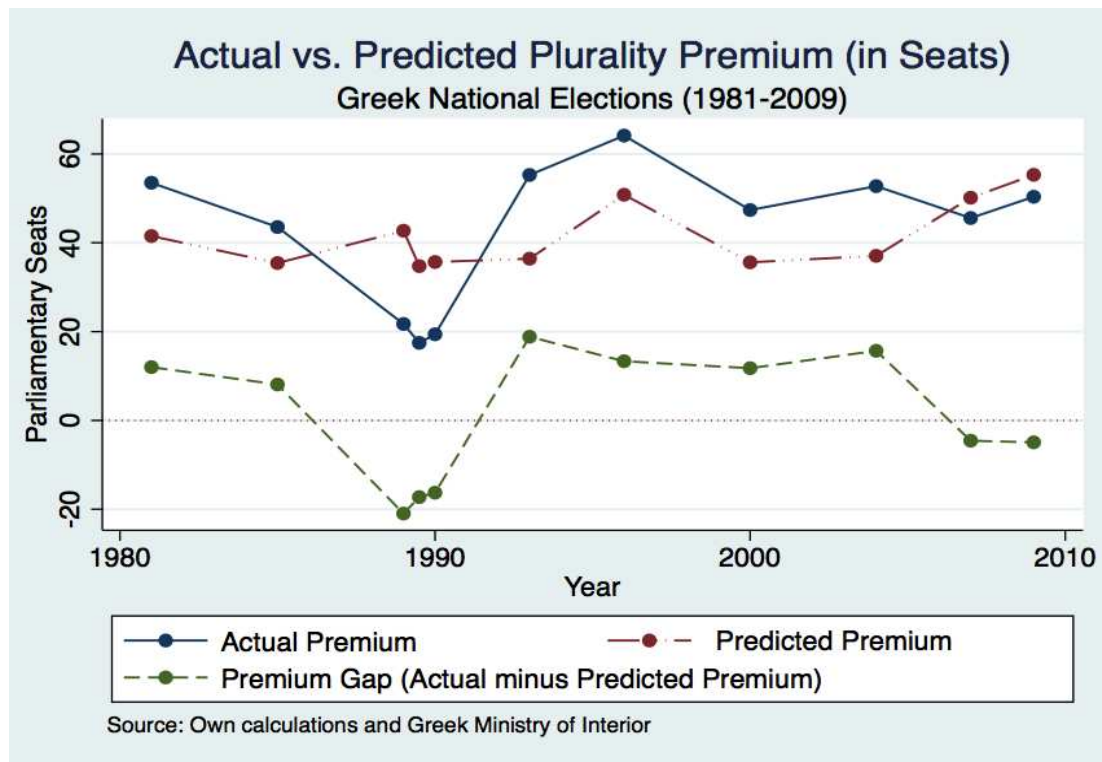
<sup>36</sup>Technically, 91.5% of the voters who participated in the referendum, following the endorsements by all major political parties in favour of the FPTP reform, approved it but, the necessary quorum for the referendum to be valid was not met (participation rate was 49.6 instead of 50%).

ian PM Renzi, leader of the biggest Italian party, has agreed with Berlusconi, the leader of the largest oppositional party and former PM, to introduce a large seat premium (bonus) that is allocated to the party (or coalition) that wins the plurality in order to guarantee its parliamentary majority. Despite some initial opposition in the Senate, the reform has cleared Italy's lower house of parliament in March and, irrespective of its final outcome, it is a clear exhibition of the incentives of the two major Italian political parties (Democratic Party/PD and Forza Italia) to collude in order to introduce a majoritarian electoral reform that will allow them to command the parliamentary majority provided that they win the election.

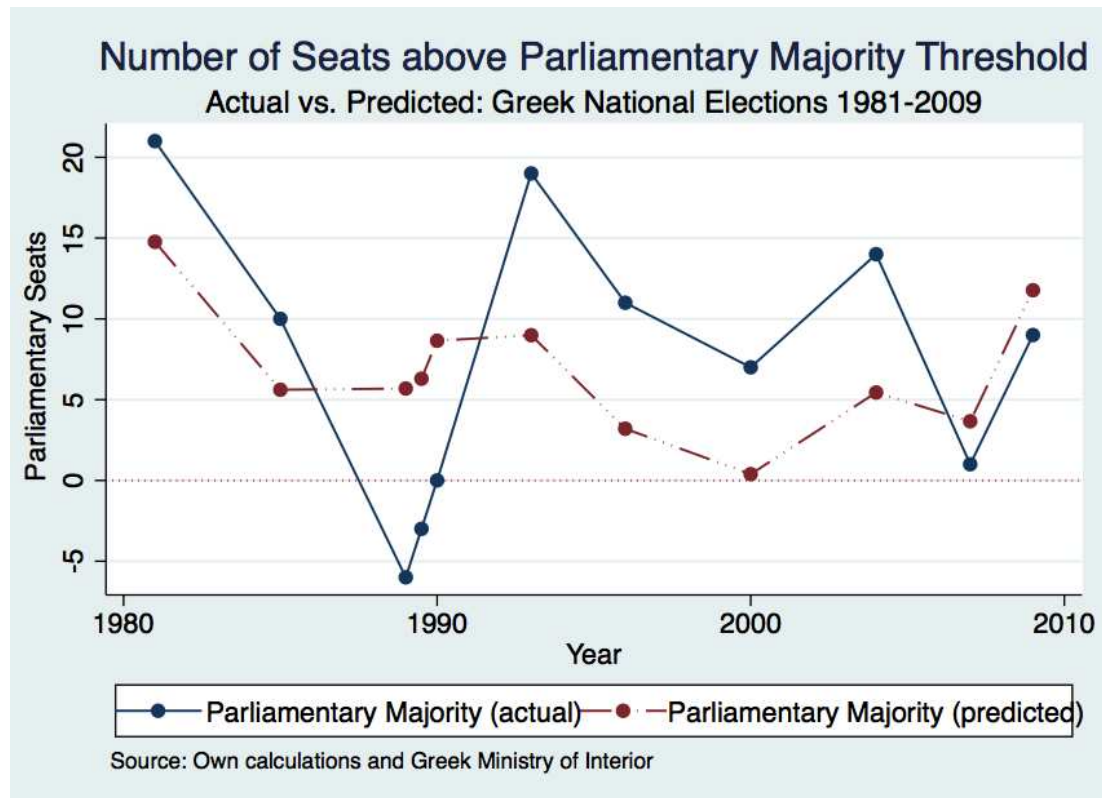
Overall, all these case-studies aimed at highlighting the exact mechanism and the strategic motivations that are behind many actual electoral reforms that took place in the last twenty years and are consistent with the theoretical framework that we have developed in this paper. While there are certainly many cases of electoral reform that our brief account here did not include, our analysis thus far has illustrated the historic and empirical relevance of our formal model.

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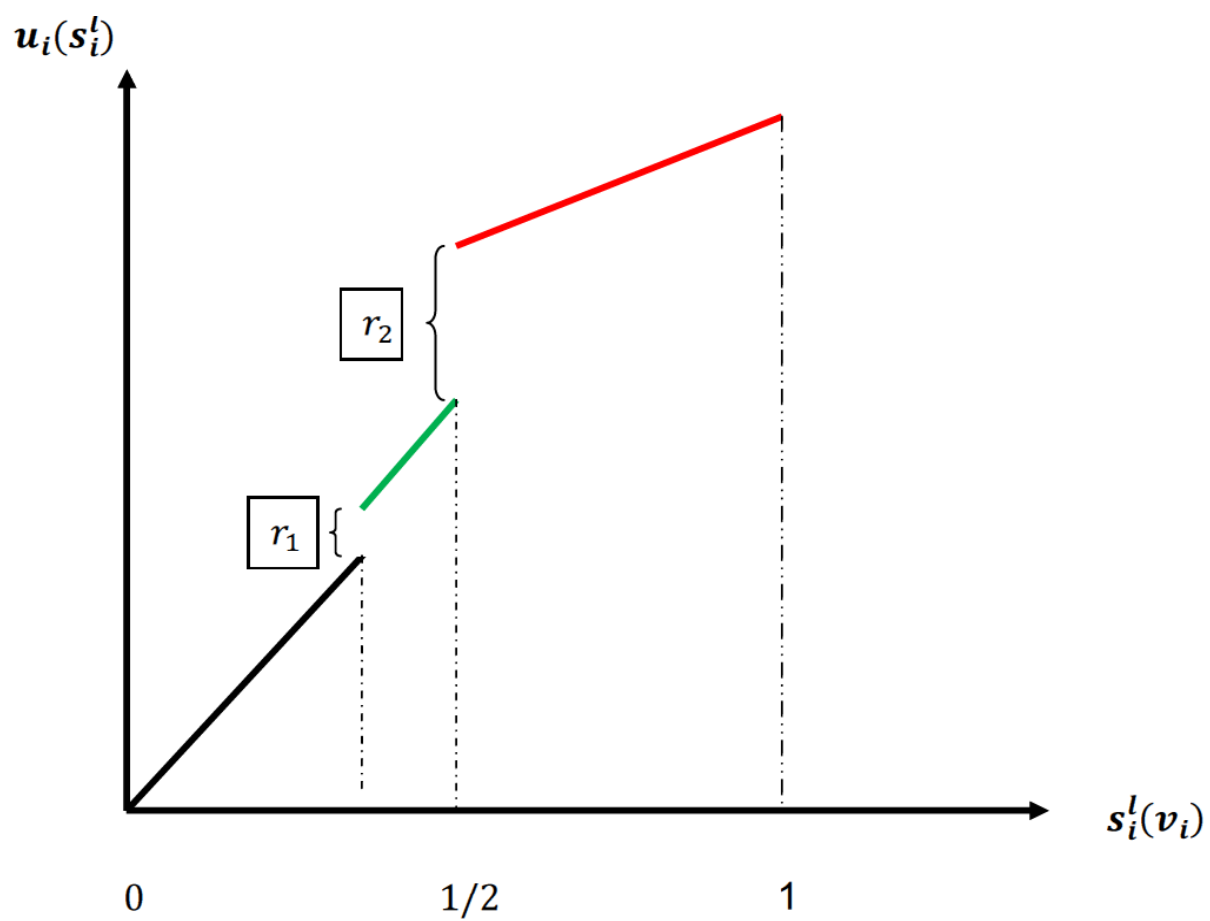


**Figure 8.** Predicted plurality premium ( $I^*$ ) and the fit with actual Greek electoral data (1981-2009).



**Figure 9.** Parliamentary majority (actual vs. predicted) and the incidence of single-party government: Greek electoral data (1981-2009).





**Figure 10.** Utility  $u_i$  for party  $i$  as a function of its seat share  $s_i$ .

TABLE IV. ESTIMATES ON THE GOODNESS OF THE FIT BETWEEN THE ACTUAL AND THE PREDICTED (EQUILIBRIUM) VALUES OF THE *LOSEMORE-HANBY* INDEX OF ELECTORAL RULE DISPROPORTIONALITY

Dependent Variable	<i>Electoral Rule Disproportionality (measured by the Losemore-Hanby index)</i>							
	OLS	OLS	FE	FE	FE	FE	FE	FE
Explanatory Variables	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Predicted <i>Losemore-Hanby</i> index	0.434 (0.145)***	-0.077 (0.103)	0.734 (0.200)***	0.170 (0.095)*	0.867 (0.224)***	0.132 (0.056)**	0.967 (0.243)***	0.199 (0.078)**
Additional controls?	Yes	Yes	No	No	No	No	Yes	Yes
Country fixed effects?	No	No	Yes	Yes	Yes	Yes	Yes	Yes
Year fixed effects?	No	No	No	No	Yes	Yes	Yes	Yes
Sample size?	$v_3 < 1/3$	Full	$v_3 < 1/3$	Full	$v_3 < 1/3$	Full	$v_3 < 1/3$	Full
$R^2$	0.10	0.05	0.28	0.06	0.55	0.24	0.60	0.29
<i>No of obs.</i>	165	302	160	302	160	302	151	285

\*  $p < 0.1$ ; \*\*  $p < 0.05$ ; \*\*\*  $p < 0.01$

*Note:* Robust standard errors clustered at the country level reported in parentheses. The predicted Losemore-Hanby index is calculated for the symmetric case. A variable indicating the *margin of electoral victory* (in %) is included as a control in all specifications throughout columns 1 to 8. The restricted sample includes the cases where there are only two dominant parties and the cumulative sum of vote shares of non-dominant parties is less than one-third (that is,  $v_3 < 1/3$ ). Other controls include: a dummy variable indicating a country with high institutional constraints (see Comparative Political Data Set I for a detailed definition), a dummy variable indicating whether a country has a tradition of coalition governments and a dummy variable indicating the type of democratic regime (parliamentary vs. presidential).

**TABLE V. Greek Legislative Elections and Optimal Majority Premium (1981-2009)**

<b><i>Election</i></b> <b><i>Year</i></b>	<b><i>Vote Shares</i></b>			<b><i>Actual Seats</i></b> <b><i>(winner)</i></b>	<b><i>Seat share</i></b> <b><i>S<sub>1</sub></i></b>	<b><i>Majority</i></b> <b><i>(actual)</i></b>	<b><i>Premium l</i></b> <b><i>(actual)</i></b>	<b><i>Majority</i></b> <b><i>(predicted)</i></b>	<b><i>Premium l*</i></b> <b><i>(predicted)</i></b>	<b><i>Premium Gap</i></b> <b><i>(l-l*)</i></b>
	V <sub>1</sub>	V <sub>2</sub>	V <sub>3</sub>							
1981	0.48	0.36	0.16	172	0.57	21	54	15	42	12
1985	0.46	0.41	0.13	161	0.54	10	44	6	35	8
1989	0.44	0.39	0.17	145	0.48	-6	22	6	43	-21
1989	0.46	0.41	0.13	148	0.49	-3	17	6	35	-17
1990	0.47	0.40	0.14	151	0.50	0	19	9	36	-16
1993	0.47	0.39	0.14	170	0.57	19	55	9	36	19
1996	0.41	0.38	0.20	162	0.54	11	64	3	51	13
2000	0.44	0.43	0.13	158	0.53	7	47	0	36	12
2004	0.45	0.41	0.14	165	0.55	14	53	5	37	16
2007	0.42	0.38	0.20	152	0.51	1	46	4	50	-5
2009	0.44	0.33	0.23	160	0.53	9	50	12	55	-5
<b><i>Average</i></b>	0.45	0.39	0.16	158.55	0.53	8	43	7	41	2